INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a 3×5-inch notecard. You may use an allowable calculator, **TI-83 or TI-84** to

- perform operations on real numbers,
- evaluate functions at specific values, and
- look at graphs and/or tables.

A TI-89, TI-Nspire, or a calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that **correct answers using incorrect reasoning may not receive any credit**. This comprehensive exam assesses your understanding of material covered throughout the entire course.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 10 problems on 12 pages. Make sure all problems and pages are present.

The exam is worth 87 points in total.

You have **1 hour** and **50 minutes** to work starting from the signal to begin. Good luck!

Final Exam Grade by Problem Number

No.	Out of	Pts.
1	10	
2	10	
3	8	
4	8	
5	16	
6	6	
7	10	
8	10	
9	6	
10	3	
Total	87	

1. (*2 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

(i) Consider the function f defined by

$$f(x) = \begin{cases} 3x^2 - 4, & x < 1 \\ 2, & x = 1 \\ 6x - 7, & x > 1 \end{cases}$$

Which of the following are true statements about this function?

I. $\lim_{x \to 1} f(x)$ exists
II. $f(1)$ exists
III. f is continuous at $x = 1$
IV. f is differentiable at $x = 1$.

(a) I only
(b) I and II only
(c) I, II, and III only
(d) I, II, and IV only
(e) I, II, III, and IV

(ii) Which of the following is equal to $\int_0^{\pi/2} \cos(x) e^{\sin(x)} dx$?

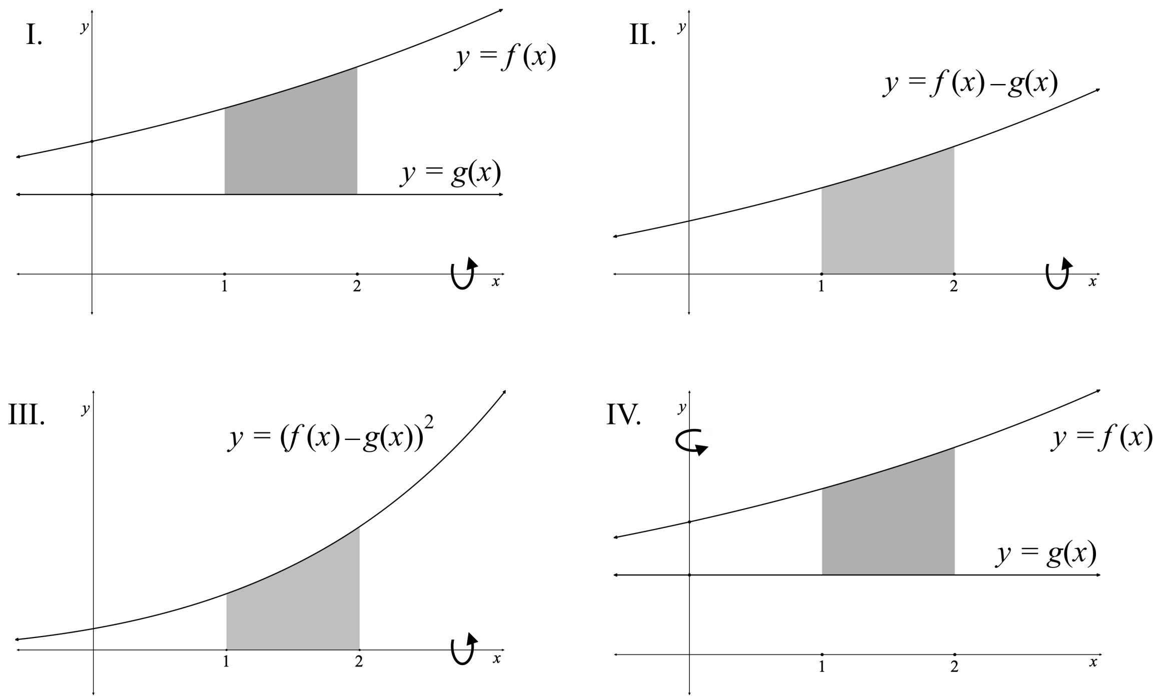
(a) $\int_0^{\pi/2} \cos(x) e^u du$
(b) $\frac{2}{\pi} \int_0^1 e^u du$
(c) $\int_0^{\pi/2} u e^{-u} du$
(d) $\int_0^1 e^u du$
(e) $\int_1^0 u e^{-u} du$

(iii) Suppose $f''(x) < 0$ for $2 \le x \le 7$. Let $y = L(x)$ represent the equation of the linear function tangent to the graph of f at the point $(2, f(2))$. Which of the following is true? (*Hint: Draw a picture.*)

(a) $L(3) > f(3)$.
(b) $L(3) < f(3)$.
(c) $L(3) = f(3)$.
(d) There is not enough information to compare $L(3)$ and $f(3)$.

(iv) The definite integral below represents the volume generated by revolving which of the bounded areas around the identified axis of rotation? (*Note that each axis of rotation is indicated by the curved arrow.*)

$$\pi \int_1^2 \left(f(x) - g(x)\right)^2 \; dx$$

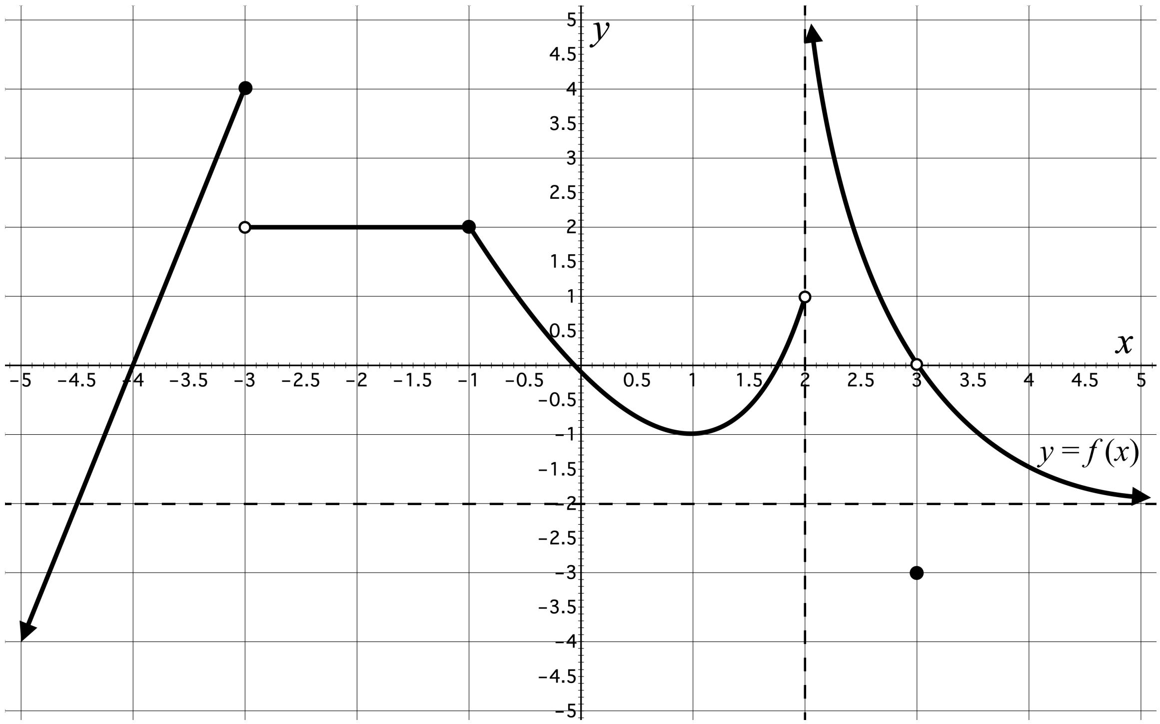


(a) I only
(b) II only
(c) III only
(d) IV only
(e) None of these

(v) If $f(x)$ varies at a constant rate of 4 with respect to x, then $\displaystyle\int_{f(x)}^{f(x+2)} 10 \, dt =$

(a) 5
(b) 20
(c) 40
(d) 80
(e) There is not enough information provided to compute this integral

2(a) (*9 points*) Answer the following questions based on the graph of $y = f(x)$ below. Assume that all critical points, points of discontinuity, and the end behavior of f can be observed from the graph below. Asymptotes are indicated by dotted lines.



Give numeric values for each of the following. Write "DNE" if the value does not exist and "∞" or "$-\infty$" as appropriate.

$\lim_{x\to -3} f(x) =$ $\qquad$ $\left.\dfrac{d^2 f}{dx^2}\right|_{x=-4.7} =$ $\qquad$ $\displaystyle\int_{-4}^{-1} f(x)\,dx =$

$f'(-3.4) =$ $\qquad$ $\lim_{x\to 2^-} f(x) =$ $\qquad$ $\lim_{\Delta x\to 0} \frac{f(1+\Delta x)-f(1)}{\Delta x} =$

$f'(-1) =$ $\qquad$ $\displaystyle\frac{d}{dx}\int_{-2}^{1} f(t)\,dt =$ $\qquad$ $\displaystyle\lim_{N\to\infty}\sum_{k=1}^{N} f\left(-2+\frac{k}{N}\right)\cdot\frac{1}{N} =$

2(b) (*1 point*) Identify a value c in the interval $(-5, 5)$ such that f is discontinuous at $x = c$ and $\lim_{x\to c} f(x)$ exists.

3(a) (*4 points*) In chemistry, pH is a scale used to measure the acidity of a solution. The pH of a solution is defined by the equation

$$\text{pH} = -\log_{10}(x)$$

where x represents the concentration of hydrogen ions. Compute the rate of change of pH with respect to hydrogen ion concentration when the pH is 2. (*Note that* pH *is a single quantity, not the product of* p *and* H.)

3(b) (*4 points*) The amount of a medication in a person's bloodstream changes at a rate of $h(t) = -700e^{-1.4t}$ mg per hour, t hours after injecting 500 mg of the medication into the bloodstream. What is the amount of medication in the person's bloodstream 3 hours after taking an initial 500 mg dose?

4. (*4 points each*) Oil leaks out of a tank at a rate of $R(t)$ gallons/hour, where t is measured in hours after the leak started. Answer each of the following with full sentences and be sure to include appropriate units.

   (a) What is the practical meaning of the statement $R'(t) < 0$ for all t?

   (b) What is the practical meaning of the statement $\int_0^2 R(t)\,dt = 500$?

5. (*4 points each*) Evaluate the following. Show all of your work and give exact answers. Do not simplify.

   (a) Let $y = \dfrac{\tan(x)}{x^2}$. Find $\dfrac{dy}{dx}$.

   (b) Let $g(t) = \sin^{-1}\left(\sqrt{t}\right)$. Find $g'(t)$.

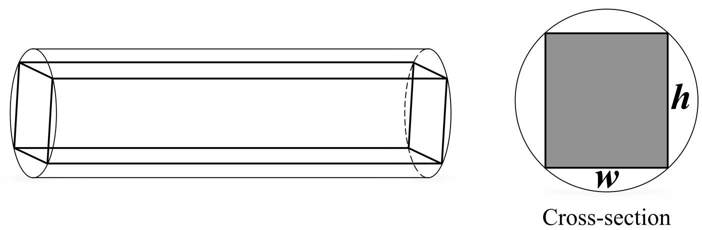
   (c) $\displaystyle\int_{\pi}^{0} (2 + \cos(\theta))\ d\theta =$

   (d) $\displaystyle\int (3x - 4)(3x^2 - 8x + 6)^7\ dx =$

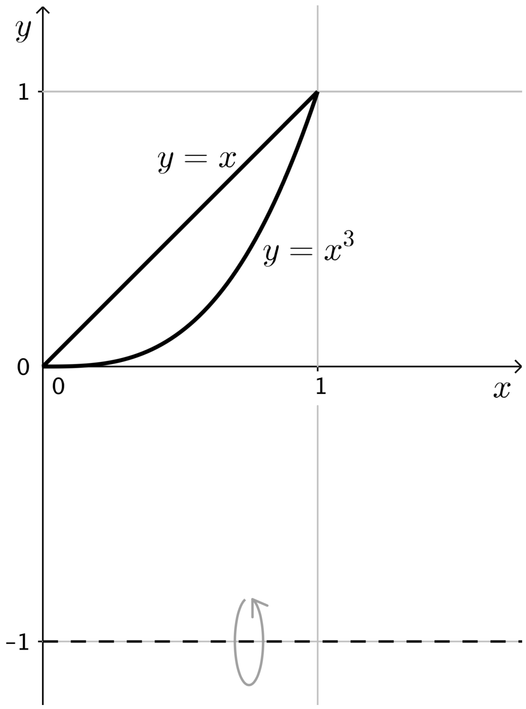
6. (*6 points*) Find the derivative $\frac{dy}{dx}$ given that x and y are related by the equation $x^3 = \sin(xy)$.

7. (*10 points*) The pressure P (in atmospheres) and volume V (in liters) in a particular balloon are related by $PV = 5$ atm L. The balloon is being squished. Currently the volume is 4 liters and is decreasing at a rate of 0.2 liters per second. At what rate is the pressure changing? Show all of your work and include units and sign.

8. (*10 points*) Suppose a rectangular beam is cut from a cylindrical log of diameter 40 cm as shown in the image below. The strength of a beam, S, is given by the formula $S = 7wh^2$, where w represents the width of the beam in centimeters and h represents the height of the beam in centimeters. Find the width and height of the beam with maximum strength that can be cut from the log. Justify your answer.



9. (*6 points*) Suppose the region bound by the graphs of $y = x$ and $y = x^3$ in the first quadrant is rotated around the line $y = -1$. Compute the volume of the resulting solid. Show all of your work.



10. (*3 points*) Write an expression in the space provided to make the equation true.

$$\int_1^e (1 - \ln x)\, dx = \int_0^1 \rule{4cm}{0.4pt} \; dy$$

BASIC FORMULAS

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$\frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$

$$\frac{d}{dx}a^x = a^x\ln(a)$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int e^u\,du = e^u + C$$

$$\int a^u\,du = \frac{a^u}{\ln a} + C$$

$$\int \sin(u)\,du = -\cos(u) + C$$

$$\int \cos(u)\,du = \sin(u) + C$$

$$\int \sec^2(u)\,du = \tan(u) + C$$

$$\int \csc^2(u)\,du = -\cot(u) + C$$

$$\int \sec(u)\tan(u)\,du = \sec(u) + C$$

$$\int \csc(u)\cot(u)\,du = -\csc(u) + C$$

$$\int \tan(u)\,du = \ln|\sec u| + C$$

$$\int \cot(u)\,du = \ln|\sin u| + C$$

$$\int \sec(u)\,du = \ln|\sec u + \tan u| + C$$

$$\int \csc(u)\,du = \ln|\csc u + \cot u| + C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int f(u(x))\cdot u'(x)\,dx = \int f(u)\,du$$

$$L_n = \Delta x\sum_{k=1}^{n} f(a+(k-1)\Delta x)$$

$$R_n = \Delta x\sum_{k=1}^{n} f(a+k\Delta x)$$

$$M_n = \Delta x\sum_{k=1}^{n} f\left(a+\frac{2k-1}{2}\Delta x\right)$$