MATH 1813: Review, Exam 4 Quizzes, Spring 2020

On all problems, be prepared to show your work or explain your reasoning.

Exam 4 Quiz, Part 1: Sections 10.1 to 10.3

Section 10.1: Compositions
1. If $A(r)$ is the area of a circle with radius $r$, and $V(A)$ is the volume of a cylinder with a circular base of area $A$ and height 3 feet, then give the meaning of $V(A(r))$.

2. Find two different possible formulas for each $g(x)$ and $h(x)$, given that $f(x) = g(h(x)) = \frac{1}{x^2 + 12x + 36}$. Assume $g(x) \neq x$ and $h(x) \neq x$.

3. Suppose $u(v(x)) = \frac{1}{x^2 - 1}$ and $v(u(x)) = \frac{1}{(x - 1)^2}$. Find possible formulas for $u(x)$ and $v(x)$.

4. Let $p(x) = 2x - 3$ and $q(x) = \sqrt{x} - 3$. Find $u(x)$ so that $q(x) = p(u(x))$.

Section 10.2: Inverses
5. Using the graphs of $f$ and $g$, solve each equation.
   (a) Solve $f(g(x)) = 1$ for $x$.
   (b) Solve $g(f(x)) = 5$ for $x$.
   (c) $f^{-1}(3)$
   (d) $g^{-1}(4)$
   (e) $g(f^{-1}(1))$

6. Let $r(x) = \frac{3x - 4}{2x + 5}$. Find $r^{-1}(x)$. 
7. The function $H = f(t) = 75 + 110(0.90)^t$ represents the temperature of a cup of coffee, in degrees Fahrenheit, $t$ minutes after it is poured.
(a) Find a formula for $f^{-1}(H)$.
(b) Evaluate $f^{-1}(150)$ and explain its meaning in terms of the cup of coffee. 
Round to 1 decimal.

8. Functions $f$, $g$, and $h$ are given below (represented by a table, a graph, and a formula, respectively).

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-1$</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

$h(x) = 4 - 3x^2$

Find each of the following
(a) $h(-1)$
(b) $f(0)$
(c) $g(1)$
(d) $f(g(0))$
(e) Solve $g(x) = 0$.
(f) Find $f\left(f^{-1}(5)\right)$.

Section 10.3: Combining of Functions
9. Find formulas for the following functions, given $f(x) = \sin x$ and $g(x) = x^2$.
(a) $f(x) + g(x)$
(b) $g(x)f(x)$
(c) $g(f(x))$

10. Give formulas for $f$, $g$, and $G$ so that $F(x) = f(G(x)) \cdot g(x)$. There may be more than one possible answer.

(a) $F(x) = 6xe^{3x^2}$
(b) $F(x) = -\frac{\sin(\sqrt{x})}{2\sqrt{x}}$

Section 11.1: Power Functions
11. Is the function a power function? If so, write it in the form $f(x) = kx^p$ and identify the values for $k$ and $p$.

(a) $g(x) = \frac{\sqrt[4]{16x}}{11x^{1/4}}$
(b) $h(x) = (8ex^3)(4x^{-1})$
12. Find a power function \( f(x) = kx^p \) through the two points: \((2, -1.6)\) and \((4, -12.8)\).

13. The pressure \( P \) of an enclosed gas is inversely proportional to the volume \( V \). In a spherical balloon with volume 3000 in\(^3\), the pressure is 20 lb/in\(^2\). If the volume of the balloon is increased to 4000 in\(^3\), what is the pressure of the gas? *Round to a whole number.*

14. Find each of the following.
(a) \( \lim_{x \to \infty} x^{-2} \)
(b) \( \lim_{x \to -\infty} (x^{-3} + 1) \)

Section 11.2/11.3: Polynomials

15. Find \( \lim_{x \to \infty} (-2x^3 + 7x - 5) \).

16. Find each of the following for the polynomial: \( f(x) = 3(1 - 2x)^5(x + 4)^3 \).
   (a) Leading term  
   (b) Degree of \( f \)
   (c) Zeros
   (d) As \( x \to -\infty \), \( f(x) \to \______ \).
   (e) As \( x \to \infty \), \( f(x) \to \______ \).

17. Find a polynomial of degree 4 (as shown) with \( f(0) = 4, \ f(-2) = 0, \ f(1) = 0, \) and \( f(4) = 0 \). *You may leave your answer in factored form.*

18. Use long division to determine whether \( x - 6 \) is a factor of \( h(x) = x^3 - x^2 - 29x - 6 \). If it is a factor, find all the zeros of \( h \).

19. Find a polynomial with least possible degree through the points \((-5,0), \ (2,0), \) and \((0,-1)\). *Write your answer in standard form.*
Section 11.4/11.5: Rational Functions

20. Find each of the following for the given rational functions $f$ and $g$ below.

$$f(x) = \frac{2x^2 - 4}{x^2 - 1} \quad \quad g(x) = \frac{2x - 1}{2x^2 + 11x - 6}$$

(a) Intercepts
(b) Horizontal or vertical asymptotes
(c) Determine whether there are any holes in the graph. If so, give the coordinates.
(d) Domain, in interval notation

21. Evaluate the limits.

$$\lim_{x \to \infty} \left( \frac{3x^2 - x + 1}{2x^2 + 5} \right) \quad \quad \lim_{x \to -\infty} \left( 8x^{-3} + 5x^{-1} + 1 \right)$$

22. Find a possible formula for the rational function shown. Note that $f(0) = -2$, $f(-4) = 0$, and the hole is at $(8, 6)$.

23. Find a possible formula for the rational function $g$ described below:
   - $g$ has two vertical asymptotes: one at $x = -2$ and one at $x = 3$
   - $g$ has a horizontal asymptote of $y = 0$
   - $g$ crosses the $x$-axis once, at $x = 5$
   - Passes through the point $(4, 0.5)$