Math 2144 Exam 2 **Fall 2019** **NAME:**

INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a 3×5-inch notecard. You may use an allowable calculator, **TI-83 or TI-84** to

- perform operations on real numbers,
- evaluate functions at specific values, and
- examine at graphs and/or tables.

A TI-89, TI-Nspire, or a calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that **correct answers using incorrect reasoning may not receive any credit**.
Turn off all noise-making devices and all devices with an internet connection and put them away. You are not allowed to have a cell phone out at any time during the exam. Put away all headphones, earbuds, etc.

This exam consists of 7 problems on 9 pages. Make sure all problems and pages are present.

The exam is worth 70 points in total.

You have **60 minutes** to work starting from the signal to begin. Good luck!

Exam 1 Grade by Problem Number

No.	Out of	Pts.
1	10	
2	12	
3	8	
4	12	
5	8	
6	10	
7	10	
Total	70	

Current Course Grade by Category

Category	Out of	Current
Exam 1	100%	
Exam 2	100%	
WebAssign	100%	
Quiz/HW	100%	
Overall 10 Week Grade	100%	

1. (*2 points each*) Answer the following multiple choice questions by circling your answer. **No justification or explanation is required**.

   (i) A spherical ice ball of radius r is melting in a liquid. It melts in a uniform fashion so that it remains a sphere while melting. Which of the following equations expresses the relationship between the rate of change of the volume V of the ice (with respect to time) and the rate of change of its radius r (with respect to time)?

   a. $V = \frac{4}{3}\pi r^3$
   b. $\frac{dV}{dt} = 4\pi r^2$
   c. $\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$
   d. $\frac{dV}{dt} = 4\pi \left(\frac{dr}{dt}\right)^2$
   e. $\frac{dV}{dt} = \frac{4}{3}\pi r^3 \cdot \frac{dr}{dt}$

   (ii) Suppose that f is continuous and differentiable on the interval $[1,\ 6]$. Also suppose that $f(1) = -8$ and $f'(x) \leq 4$ for all x in the interval $[1,\ 6]$. What is the largest possible value for $f(6)$?

   a. 20
   b. -4
   c. -32
   d. 30
   e. 12

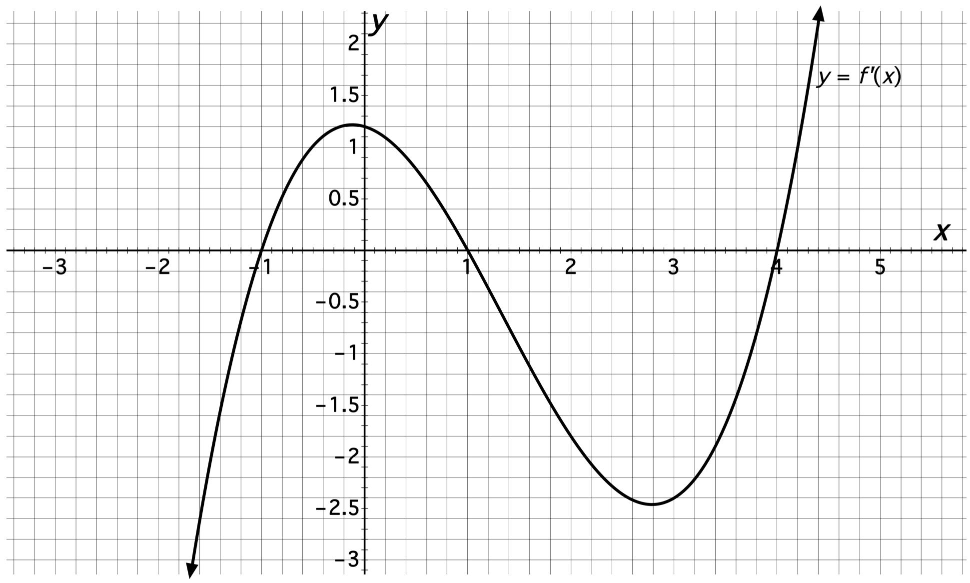
   (iii) Assume that g is twice-differentiable on the closed interval $[1,\ 5]$ and has one critical point at $x = 2.5$. Circle the statement(s) that, if true, would be a valid justification that $g(2.5)$ is a maximum of g on this interval.

   $g'(2.5) = 0$ $\qquad$ $g''(2.5) = 0$ $\qquad$ $g'(2.5) > 0$ $\qquad$ $g''(2.5) > 0$

   $g'(2.5) < 0$ $\qquad$ $g''(2.5) < 0$ $\qquad$ $g'(1) > 0$ and $g'(3) < 0$

   $g'(1) < 0$ and $g'(3) > 0$ $\qquad$ $g(1) < g(2.5) < g(5)$ $\qquad$ $g(1) < g(5) < g(2.5)$

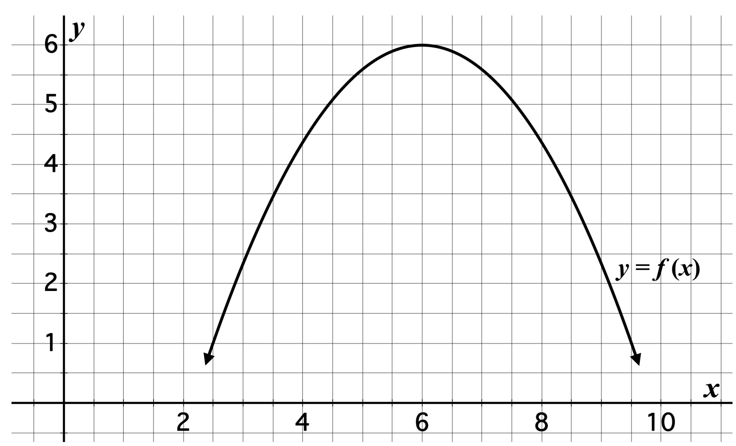
(iv) The ***derivative*** function $y = f'(x)$ is graphed below.



Based on this graph, the function $y = f(x)$ is *increasing* on the interval(s)?

a. $(-\infty,\ -1)$
b. $(-1,\ 1)$ and $(4,\ \infty)$
c. $(-\infty,\ -0.2)$ and $(2.8,\ \infty)$
d. $(-\infty,\ -1)$ and $(1,\ 4)$
e. $(4,\ \infty)$

(v) The graph of a twice-differentiable function f is shown below. Which of the following is true?



(a) $f(6) < f'(6) < f''(6)$

(b) $f'(6) < f(6) < f''(6)$

(c) $f''(6) < f'(6) < f(6)$

(d) $f(6) < f''(6) < f'(6)$

(e) $f''(6) < f(6) < f'(6)$

2. (*12 points*) Use calculus to determine the following for the function

$$f(x) = 4xe^{3x}.$$

(a) (*4 points*) Find all critical points of f. Work must be shown to receive credit.

(b) (*4 points*) Find all inflection points of f. Work must be shown to receive credit.

(c) (*4 points*) Determine the absolute maximum and absolute minimum value of $f(x)$ on the interval $[-2, 0]$. Work must be shown to receive credit.

3. (*4 points each*) Evaluate the following limits or state that they do not exist ("DNE"). Use ∞ or $-\infty$ if either is appropriate. Numbers alone without justification (either algebra and/or quoting theorems where applicable) will receive no credit. **If you use L'Hôpital's Rule, clearly indicate when you do so**.

(a) $\lim_{x \to 1} \dfrac{3x^2 + 4x - 7}{x^3 - 8x^2 + 2x + 5} =$

(b) $\lim_{x \to 1} \dfrac{3x^2 - 4x + 7}{x^3 + 8x^2 - 2x + 5} =$

4. (*4 points each*) Compute the following derivatives. Do not simplify.

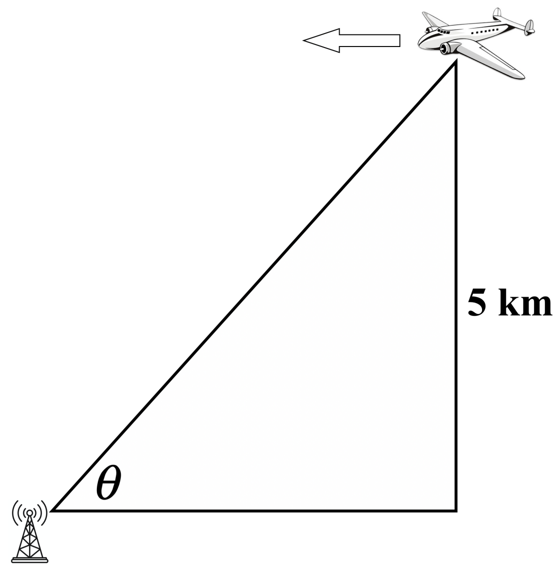
   (a) Let $f(x) = \ln\left(\sqrt[4]{x^3}\right)$. Find $f'(x)$.

   (b) Let $y = \arctan(x + e^x)$. Find $\dfrac{dy}{dx}$.

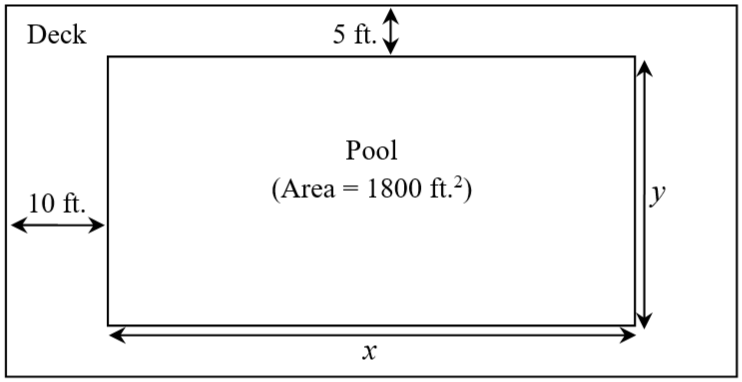
   (c) Let $g(\theta) = \sec(\pi\theta)$. Find $g''(\theta)$. (*Note that* $g''(\theta)$ *is the same as* $\dfrac{d^2g}{d\theta^2}$.)

5. (*8 points*) Find $\dfrac{dy}{dx}$ for the curve $x\ln(y) + y^3 = \ln(x)$. Your answer for $\dfrac{dy}{dx}$ should contain both x and y.

6. (*10 points*) An airplane flying at a constant speed of 460 km/hr at an altitude of 5 km is approaching a position directly over a radar station. Let θ represent the measure of the angle of elevation from the radar station to the plane as shown in the diagram below. Determine the rate at which the angle of elevation is changing at the moment $\theta = \pi/3$.



7. (*10 points*) A rectangular swimming pool is to be built with an area of 1,800 square feet. The owner wants 5-foot wide decks along two sides of the pool and 10-foot wide decks at the two ends as shown in the picture below. Find the dimensions of the smallest possible combined area of the pool and deck satisfying these conditions. **Justify your response.**



BASIC FORMULAS

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$\frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$

$$\frac{d}{dx}a^x = a^x\ln(a)$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$