MATH 1813: Final Exam Review, Fall 2019

Note: Problem numbers correspond to the review where the problem was originally assigned.

<u>Exam 1 Review</u>

Section 1.2: Rates of Change

2. A survey of the U.S. population is done every 10 years when the census is taken. The approximate population N of the U.S., in millions, is a function of the date d.

d = date	1970	1980	1990	2000	2010
N = Pop. of U.S. (in million	203	227	249	281	309

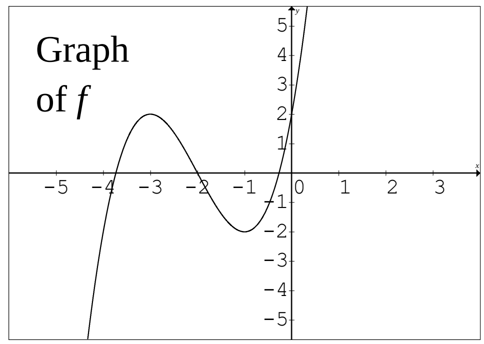
(a) What was the *average rate of change* in the U.S. population from 1990 to 2000? What does this number mean?

(b) What was the *average rate of change* in the U.S. population from 2000 to 2010?

3. Find the intervals on which the function f is *increasing* or *decreasing*.

Increasing: ________________

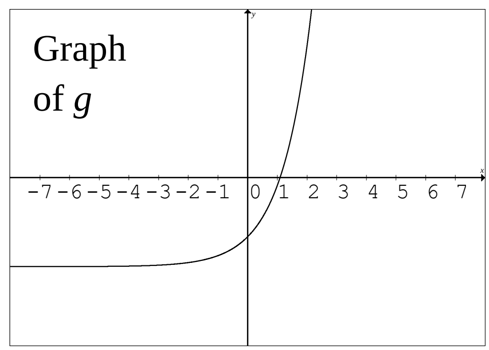
Decreasing: ________________



4. For the function g shown, on which interval of x is the average rate of change of g the greatest? Why?

(a) –7 to –5 (b) –5 to –2

(c) –2 to 1 (d) 1 to 2



Section 1.3/1.4: Linear Functions

5. The temperature at which water boils depends on the altitude above sea level, as shown in the table.

a = Altitude, in thousands of feet	1.1	2.2	3.3	4.4
T = Temperature at which water boils, in °F	210	208	206	204

(a) Is the data in the table linear? Why?

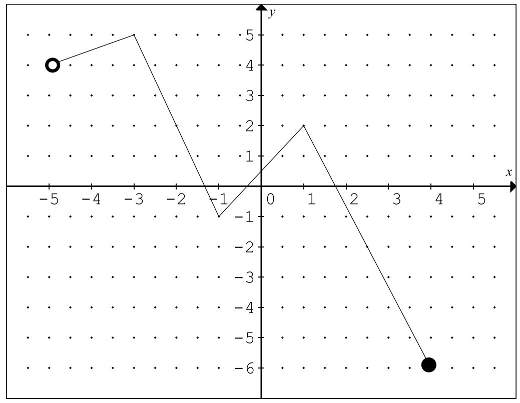
(b) Use a formula to express the temperature T as a function of the altitude a.

(c) Give the slope of the function and explain its meaning in practical terms.

(d) At what temperature does water boil in Stillwater, which has an altitude of about 900 feet?

Section 2.1/2.2: Input/Output, Domain/Range

9. Find each of the following for the function f shown.



(a) $f(-3)$ (b) $f(4)$

(c) For what value(s) of x is $f(x) = 2$?

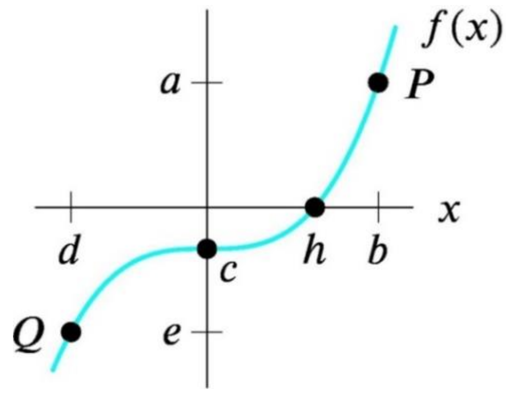
(d) For what value(s) of x is $f(x) > 2$?

(e) Find the domain and range for the function.

10. Suppose $f(x) = 3 - x^2$. Find the following, and simplify completely.

$$\frac{f(a+h) - f(a)}{h}, \text{ where } h \neq 0$$

11. Use the letters a, b, c, d, e, and h in the figure below to answer the following questions.



(a) What are the coordinates of points P and Q?

(b) Evaluate $f(b)$.

(c) Solve $f(x) = e$ for x.

(d) Suppose $c = f(z)$ and $z = f(x)$. What is x?

(e) Suppose $f(b) = -f(d)$. What additional information does this give you?

12. Find the domain for each function. *Write your answer in interval notation.*

(a) $f(x) = 3x^2 - 8x + 2$ (b) $f(x) = \frac{\sqrt{x+5}}{x+1}$

Section 2.6: Concavity

16. Calculate successive rates of change for the function $g(t)$ in the table below to decide whether you expect the graph of $g(t)$ to be concave up or concave down (without graphing).

t	3	3.6	4.2	4.8	5.4
$g(t)$	–8	–5	–3	–2	–1.5

18. Sketch a possible graph of a function $f(x)$ with all of these properties:

- $f(0) = 3$
- The solution to $f(x) = 0$ is $x = -2$
- f is decreasing and concave up for $-\infty < x < -2$
- f is increasing and concave up for $-2 < x < 1$
- f is increasing and concave down for $1 < x < 6$
- f is decreasing and concave down for $6 < x < \infty$

Sections 3.1 and 3.2: Quadratic Functions

19. What is a formula for the quadratic function described in the table? *Write your answer in standard form.*

x	–5	0	2	4	6
$f(x)$	0	–15	–14	–9	0

20. Find a formula for a quadratic function with vertex at (–6, 1) and passing through the point (–2, 8). *Write your answer in vertex form.*

<u>Exam 2 Review</u>

Section 4.1: Exponential Functions

1. The population of India was about 1.22 billion people in 2013 and was growing at a rate of about 1.28% per year.

(a) Write a formula for the population P of India, in billions, as a function of t years since 2013.

(b) If the growth rate stays constant, predict the population of India in the year 2020. *Round to 2 decimals.*

(c) Use your formula to find the average rate of change of India's population from 2013 to 2015. *Round to 4 decimals.*

(d) Using your formula, by what percent did the population of India increase in the two-year period between 2013 and 2015?

Section 4.2: Linear vs. Exponential

2. The table shows values for a linear function and an exponential function.

x	3	6	9	12
$f(x)$	5.12	3.2	2	1.25
$g(x)$	3.05	2.45	1.85	1.25

(a) Which function is *linear* and which is *exponential*? Why?

(b) Find possible formulas to represent the two functions f and g. *Round parameters to four decimals.*

Section 4.3: Exponential Graphs

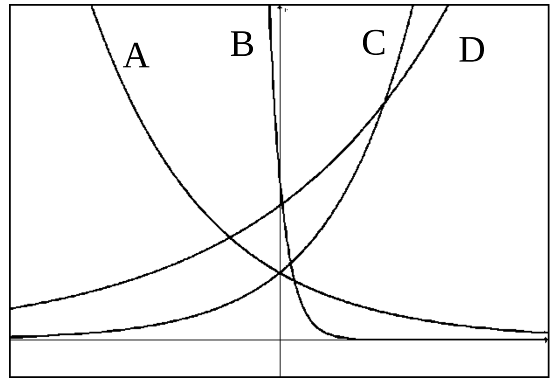
5. Assume the equations for A, B, C, and D can all be written in the form $y = ab^t$.

(a) Which function has the largest value for *a*?

(b) Which two functions have the same value for *a*?

(c) Which function has the smallest value for *b*?

(d) Which function has the largest value for *b*?



Sections 4.4/4.5: The Number e

8. Suppose 2 mg of a drug is injected into a person's bloodstream. As the drug is metabolized, the quantity diminishes at the continuous rate of 4% per hour.

(a) Find a formula for $Q(t)$, the quantity of the drug remaining in the body after *t* hours, in the form $Q(t) = a \cdot e^{kt}$.

(b) Find a formula for $Q(t)$ in the form $Q(t) = a \cdot b^t$. *Round the decay factor to 4 decimals.*

(c) By what percent does the drug level decrease during any given hour? *Round to 2 decimals.*

(d) By what percent does the drug level decrease in 5 hours? *Round the percent to 2 decimals.*

Section 5.1: Logarithms

9. Solve for *x* exactly. Then, *round your answer to 2 decimals.*

(a) $\log(x+3) - \log(x-1) = 1$ (b) $6(1.2)^x = 5(1.7)^x$

10. Solve for *x*. *Give you answer in exact form.*

(a) $3e^{2x} + 5e^{2x} = 7$ (b) $\log(4x+5) \cdot \log(3x^2+1) = 0$

Section 5.2: Logs and Exponentials

11. If the price of a gallon of milk climbs at a continuous rate of 3% annually, how many years will it take its price to double? *Round to 2 decimals.*

12. Plutonium-238 has a half-life of 87.7 years. How long will it take a sample of plutonium-238 to decrease to 20% of its original mass? *Use the model* $f(t) = ae^{kt}$ *and do not round your calculated value of k. Round the final answer to a whole number.*

Section 5.3: Graphs of Logarithms

14. Graph the function $y = \ln(x+3)$. Identify any vertical asymptotes. State the domain and range of the function.

Section 6.1: Shifts and Reflections

17. A function $f(x)$ has domain $[-2,15)$ and range $[12,25)$. Find a possible formula for $h(x)$ in terms of $f(x)$, if the domain of $h(x)$ is $[-5,12)$ and the range is $(-25,-12]$.

Sections 6.2/6.3: Other Transformations (Stretches & Shrinks)

21. The point (–6, 7) lies on the graph of $g(x)$. What point lies on the graph of $y = 2g(x-3)+4$?

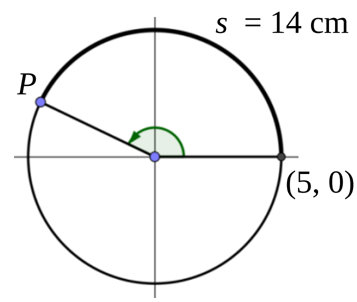
Section 7.2: Sine and Cosine

26. Find angles between 0° and 360° that have the same

(a) cosine as 230° (b) sine as 305°

Section 7.3: Radians and Arc Length, Unit Circle

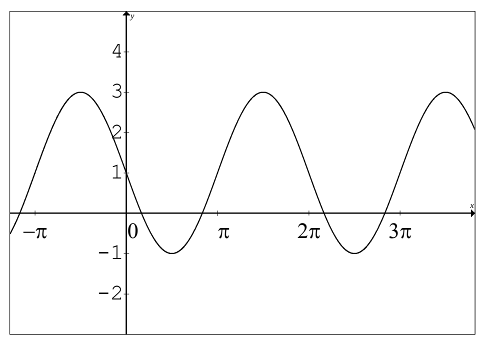
29. *P* is a point on a circle of radius 5 cm. The length of the arc from (5, 0) to *P* is 14 cm. What are the coordinates of point *P*? *Round to 2 decimals.*



<u>**Exam 3 Review**</u>

Section 7.4: Sine and Cosine Graphs

1. Determine the amplitude, period, midline, and whether the graph has an *x*-axis reflection and/or a vertical shift. Then, write a possible equation for the graph.

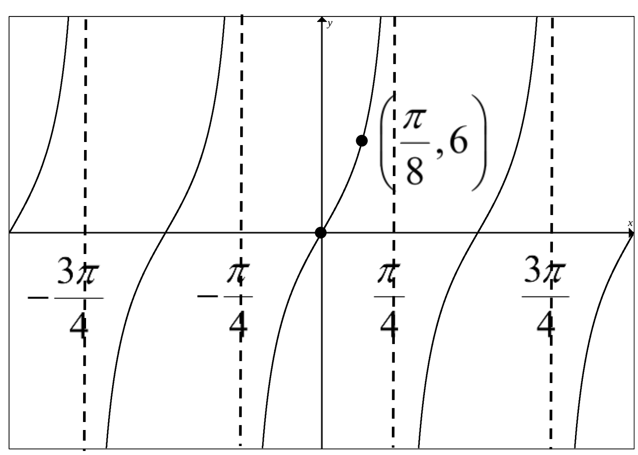


Section 7.5: Sinusoidal Functions

5. A population of animals oscillates between a low of 1300 on January 1 ($t = 0$) and a high of 2200 on July 1 ($t = 6$). Assume the population can be modeled by a sinusoidal function. Find a formula for the population *P* in terms of the time *t* in months.

Section 7.6: Tangent

8. Given the tangent function that passes through the points (0, 0) and $(\pi/8, 6)$, and with asymptotes as shown in the figure below, first **describe** in words how this graph is related to the graph of $y = \tan x$. Then, find a **possible formula** of the form $f(t) = A\tan(Bt) + C$.



Section 7.7: Other Trig Functions, Identities

10. Simplify the expression (for values of the variable for which it is defined). Remember to show each of your steps in a clearly shown order. *Write your final answer in simplest terms, using a single trigonometric function (if possible).*

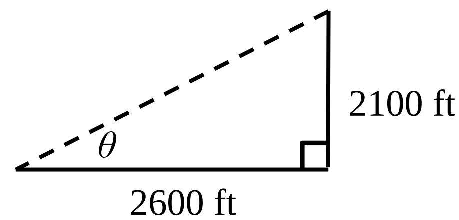
(a) $\tan\theta\csc\theta$ (b) $(\cos x+\sin x)^2-2\cos x\sin x$

12. Use *identities* to find the **exact values** of each of the following, given that $\tan\theta=\frac{5}{2}$ and $\pi\le\theta\le\frac{3\pi}{2}$.

(a) $\cos\theta$ (b) $\sin\theta$ (c) $\sec\theta$

Section 7.8: Inverse Trig Functions

15. If a ski slope has the measurements as shown below, what is its angle of elevation, in degrees? *Round your answer to the nearest tenth of a degree.*

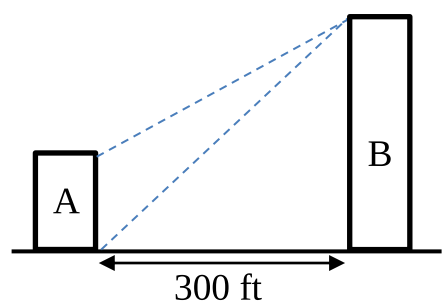


18. Solve the equation for a value of θ in the first quadrant. Give your answer in radians and degrees. *Round radian angles to 3 decimals and round degree angles to 1 decimal*.

$8\cos\theta-5=0$

Section 8.1: Right Triangle Trigonometry

24. The angle of elevation from the bottom of building A to the top of building B is 48°. When standing on the roof of building A, the angle of elevation to building B is 32°. If the two buildings are 300 feet apart, how tall is building A? *Round to the nearest foot.*



Section 9.1: Trig Equations

25. Find approximate solutions to $\cos\theta=-0.43$ on the interval $-2\pi\le\theta\le 2\pi$. *Round to 2 decimals.*

26. Find the exact solutions to $5(\sin\theta-1)=3\sin\theta-7$ on the interval $0\le\theta<2\pi$.

Section 10.1: Compositions

30. Find possible formulas for $g(x)$ and $h(x)$, given that $f(x)=g(h(x))=\frac{1}{x^2+12x+36}$.

31. Suppose $u(v(x)) = \frac{1}{x^2 - 1}$ and $v(u(x)) = \frac{1}{(x-1)^2}$. Find possible formulas for $u(x)$ and $v(x)$.

Section 10.2: Inverses

35. The function $H = f(t) = 75 + 110(0.90)^t$ represents the temperature of a cup of coffee, in degrees Fahrenheit, *t* minutes after it is poured.

(a) Find a formula for $f^{-1}(H)$.

(b) Evaluate $f^{-1}(150)$ and explain its meaning in terms of the cup of coffee. *Round to 1 decimal.*

Section 10.3: Combining of Functions

38. Give formulas for *f*, *g*, and *G* so that $F(x) = f(G(x)) \cdot g(x)$. *There may be more than one possible answer.*

(a) $F(x) = 6xe^{3x^2}$ (b) $F(x) = -\frac{\sin\left(\sqrt{x}\right)}{2\sqrt{x}}$

<u>New Material</u>

Section 11.1: Power Functions

1. Is the function a power function? If so, write it in the form $f(x) = kx^p$ and identify the values for *k* and *p*.

(a) $g(x) = \frac{\sqrt{16x}}{11x^{1/4}}$ (b) $h(x) = (8ex^3)(4x^{-1})$

2. Find a power function $f(x) = kx^p$ through the two points: $(2, -1.6)$ and $(4, -12.8)$.

3. The pressure *P* of an enclosed gas is inversely proportional to the volume *V*. In a spherical balloon with volume 3000 in^3, the pressure is 20 lb/in^2. If the volume of the balloon is increased to 4000 in^3, what is the pressure of the gas? *Round to a whole number.*

4. Find each of the following.

(a) $\lim\limits_{x\to\infty} x^{-2}$ (b) $\lim\limits_{x\to-\infty} (x^{-3} + 1)$

Section 11.2: Polynomials

5. Find $\lim\limits_{x\to\infty}(-2x^3+7x-5)$.

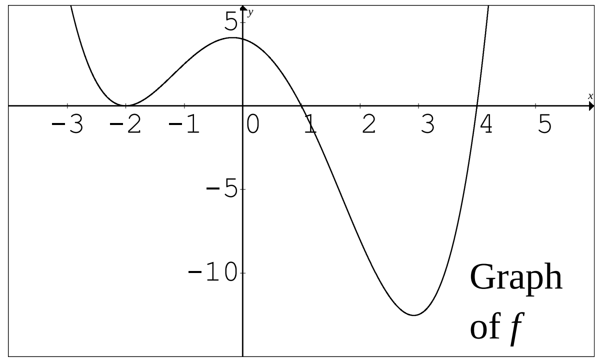
6. Find each of the following for the polynomial: $f(x)=3(1-2x)^5(x+4)^3$.

(a) Leading term (b) Degree of *f* (c) Zeros

(d) As $x\to-\infty$, $f(x)\to$_______. (e) As $x\to\infty$, $f(x)\to$ _______.

Section 11.3: Short-Run Behavior of Polynomials

7. Find a polynomial of degree 4 (as shown) with $f(0)=4$, $f(-2)=0$, $f(1)=0$, and $f(4)=0$. *You may leave your answer in factored form.*



Graph of *f*

8. Use long division to determine whether $x-6$ is a factor of $h(x)=x^3-x^2-29x-6$. If it is a factor, find all the zeros of *h*.

9. Find a polynomial with least possible degree through the points $(-5,0)$, $(2,0)$, and $(0,-1)$. *Write your answer in standard form.*

Section 11.4: Rational Functions

10. Find each of the following for the given rational functions *f* and *g* below.

$f(x)=\dfrac{2x^2-4}{x^2-1}$ $g(x)=\dfrac{2x-1}{2x^2+11x-6}$

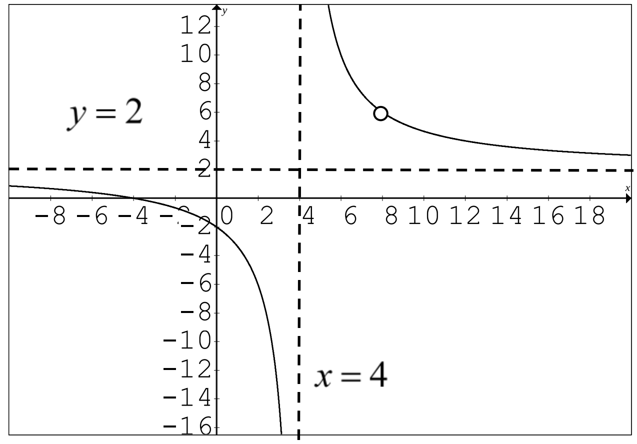
(a) Intercepts
(b) Horizontal or vertical asymptotes
(c) Determine whether there are any holes in the graph. If so, give the coordinates.
(d) Domain, in interval notation

11. Evaluate the limits.

(a) $\lim\limits_{x\to\infty}\left(\dfrac{3x^2-x+1}{2x^2+5}\right)$ (b) $\lim\limits_{x\to-\infty}\left(8x^{-3}+5x^{-1}+1\right)$

Section 11.5: Short-Run, Rational Functions

12. Find a possible formula for the rational function shown. Note that $f(0) = -2$, $f(-4) = 0$, and the hole is at (8, 6).



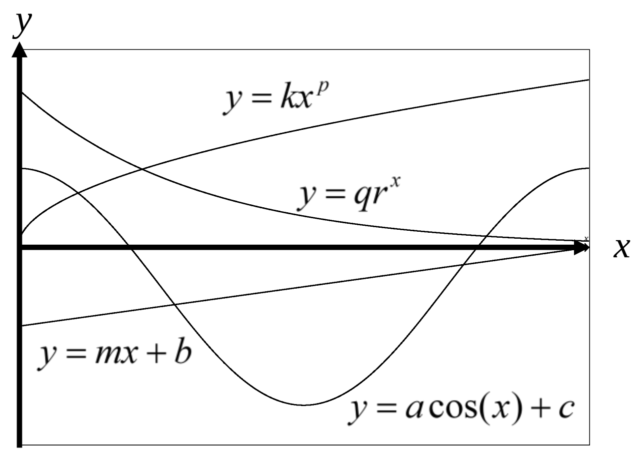
13. Find a possible formula for the rational function g described below:

- g has two vertical asymptotes: one at $x = -2$ and one at $x = 3$
- g has a horizontal asymptote of $y = 0$
- g crosses the x-axis once, at $x = 5$
- Passes through the point (4, 0.5)

Mixed Sections

14. The graphs of four different functions are shown, defined in terms of eight constants: a, b, c, k, m, p, q, and r. Use the graphs to answer the following questions. Justify your answers with appropriate explanations.

(a) Which of the constants are definitely negative?
(b) Which of the constants are definitely positive?
(c) Which of the constants are definitely greater than zero, but less than one?



15. Let f be a sine function with period 4π, amplitude 3, and midline $y = 2$. Find each of the following.

(a) Find the **amplitude** and **midline** of $g(x) = 2f(x) + 1$.

(b) If $p(x) = \frac{1}{3}x$, what is the **period** of $r(x) = f(p(x))$?