

**INSTRUCTIONS:** This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a  $3 \times 5$ -inch notecard. You may use an allowable calculator, **TI-83 or TI-84** to

- perform operations on real numbers,
- evaluate functions at specific values, and
- examine at graphs and/or tables.

A TI-89, TI-Nspire, or a calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that **correct answers using incorrect reasoning may not receive any credit**.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 7 problems on 8 pages. Make sure all problems and pages are present.

The exam is worth 70 points in total.

You have **60 minutes** to work starting from the signal to begin. Good luck!

**Exam 1 Grade by  
Problem Number**

No.	Out of	Pts.
1	10	
2	11	
3	15	
4	8	
5	10	
6	10	
7	6	
Total	70	

**Current Course Grade by Category**

Category	Out of	Current
Exam 1	100%	
Exam 2	100%	
WebAssign	100%	
Quiz/HW	100%	
Overall 10 Week Grade	100%	

1. (2 points each) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

- (i) A cylindrical container of fixed radius  $r$  is being filled with water. Which of the following equations expresses the relationship between the rate of change of the volume  $V$  of the water in the container (with respect to time) and the rate of change of the height  $h$  of the water in the container (with respect to time)?

- a.  $\frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt}$
- b.  $V = \pi r^2 h$
- c.  $\frac{dV}{dt} = 2\pi r h + \pi r^2 \cdot \frac{dh}{dt}$
- d.  $V = 2\pi r h$
- e.  $\frac{dh}{dt} = \pi r^2 h$

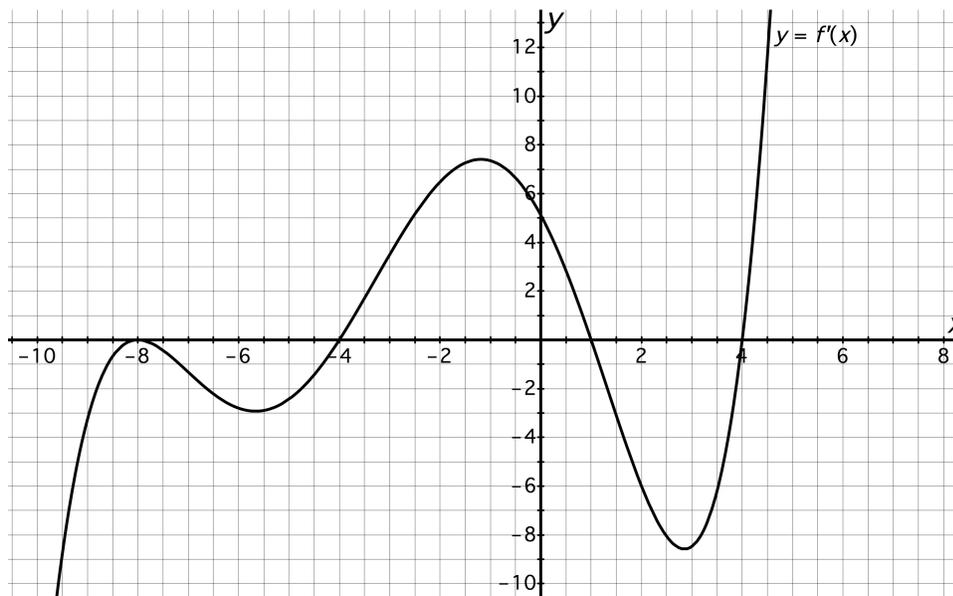
- (ii) Suppose  $f'(c) = 0$  and  $f''(x) < 0$  for all  $x$  around  $c$ . Which of the following is true?

- a.  $f(c)$  is an inflection point.
- b.  $f(c)$  is a local minimum.
- c.  $f(c)$  is a local maximum.
- d.  $f'(c)$  is positive.
- e.  $f'(c)$  is negative.

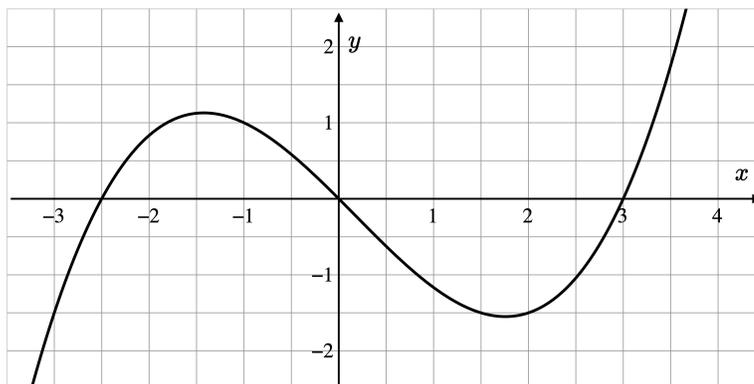
- (iii) Suppose the function  $g(t) = 6t^2 - 0.75^3$  represents the height of a golfball above the ground (in yards)  $t$  seconds since the ball was struck (only for values of  $t$  between 0 and 8). By solving which of the following equations can we determine the value of  $t$  for which the height of the ball above the ground is maximized?

- a.  $g'(t) = 12t - 2.25t^2$
- b.  $0 = 12t - 2.25t^2$
- c.  $g(t) = 6t^2 - 0.75^3$
- d.  $0 = 6t^2 - 0.75^3$
- e.  $g''(t) = 12 - 4.5t$

- (iv) Below is a graph of a derivative function  $y = f'(x) = \frac{df}{dx}$ . Based on this graph, for what value(s) of  $x$  does  $f(x)$  have a local minimum value?



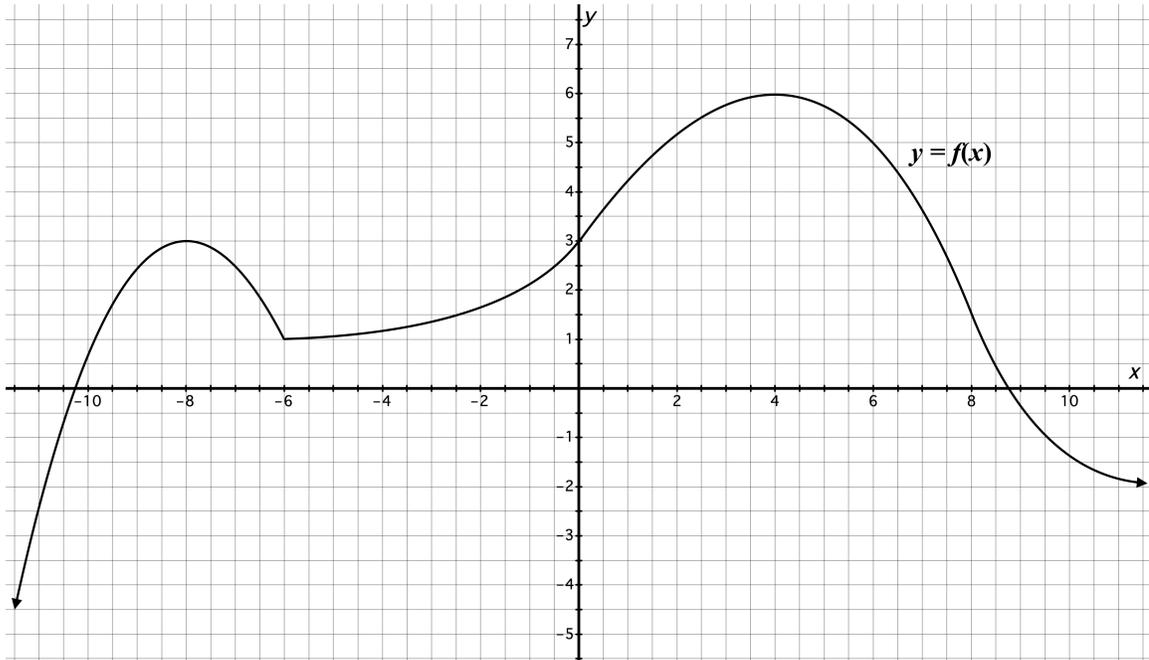
- a.  $x = -5.7$  and  $x = 2.8$
  - b.  $x = -4$  and  $x = 4$
  - c.  $x = 1$
  - d.  $x = -8$ ,  $x = -4$ ,  $x = 1$ , and  $x = 4$
  - e.  $f(x)$  does not have a minimum value.
- (v) Consider the following graph of  $f(x) = x \sin(x)$  on the domain  $[-3, 3]$ . How many values of  $c$  in  $(-3, 3)$  appear to satisfy the Mean Value Theorem equation  $f'(c) = \frac{f(b) - f(a)}{b - a}$  where  $a = -3$  and  $b = 3$ .



- a. None
- b. One
- c. Two
- d. Three
- e. Four or more

**Math 2144 Exam 2**

2. (11 points) The graph of the function  $f$  is shown below. Interpret the graph as it looks to answer the following questions.



- (a) (6 points) Use the above graph to fill in the following table with the signs of  $f'(x)$  and  $f''(x)$  on the indicated intervals. Use a “+” sign to indicate the values are always positive and a “-” sign to indicate the values are always negative.

	$(-10, -8)$	$(-8, -6)$	$(-6, -0)$	$(0, 4)$	$(4, 8)$	$(8, 10)$
Sign of $f'(x)$						
Sign of $f''(x)$						

- (b) (3 points) List the  $x$ -coordinates of all critical points.

- (c) (2 points) List the coordinates of all inflection points.

3. (5 points each) Compute the following derivatives. Do not simplify.

(a) Let  $f(\theta) = 4^{\cos(\theta)}$ . Find  $f'(\theta)$ .

(b) Let  $y = e^x \cdot \sec(x)$ . Find  $\frac{dy}{dx}$ .

(c) Let  $g(t) = \ln(t^6)$ . Find  $g''(t)$ .

4. (8 points) Find  $\frac{dy}{dx}$  for the curve  $x^4 - \cos(y) = \sin(x)$ . Your answer for  $\frac{dy}{dx}$  should contain both  $x$  and  $y$ .

5. (10 points) Use calculus to determine the following for the function

$$f(x) = x^3 - 9x^2 - 48x + 18.$$

(a) (6 points) Find the  $x$ -coordinates of all critical points of  $f$ . Work must be shown to receive credit.

(b) (4 points) Determine the absolute maximum and absolute minimum value of  $f(x)$  on the interval  $[-5, 15]$ . Work must be shown to receive credit.

6. (10 points) A stone thrown into a pond produces a circular ripple. If the radius of the ripple increases at a rate of 1.5 ft/sec, how fast is the area growing when the radius is 8 feet? State your answer as an exact value (i.e., in terms of  $\pi$ ).

7. (6 points) Suppose that  $f$  is continuous and differentiable on the interval  $[3, 8]$ . Also suppose that  $f(3) = 2$  and  $f'(x) \leq 2$  for all  $x$  in the interval  $[3, 8]$ . What is the largest possible value for  $f(8)$ ? Justify your response.