MATH 1813: Exam 2 Review, Fall 2019

On all problems, show your work or explain your reasoning.

Section 4.1: Exponential Functions

1. The population of India was about 1.22 billion people in 2013 and was growing at a rate of about 1.28% per year.

(a) Write a formula for the population P of India, in billions, as a function of t years since 2013.

(b) If the growth rate stays constant, predict the population of India in the year 2020. *Round to 2 decimals.*

(c) Use your formula to find the average rate of change of India's population from 2013 to 2015. *Round to 4 decimals.*

(d) Using your formula, by what percent did the population of India increase in the two-year period between 2013 and 2015?

Section 4.2: Linear vs. Exponential

2. The table shows values for a linear function and an exponential function.

x	3	6	9	12
$f(x)$	5.12	3.2	2	1.25
$g(x)$	3.05	2.45	1.85	1.25

(a) Which function is *linear* and which is *exponential*? Why?

(b) Find possible formulas to represent the two functions f and g. *Round parameters to four decimals.*

3. The U.S. is the one of the world's largest producers of wheat. In 2000, the U.S. produced 60.6 million metric tons. In 2014, it produced 55.4 million metric tons.

(a) Find a formula for $f(t)$ assuming linear growth. Interpret the slope of your formula in terms of the wheat production. *Round the slope to 3 decimals.*

(b) Find a formula for $g(t)$ assuming exponential growth or decay. Interpret the parameters of your formula in terms of the wheat production. *Round the growth or decay factor to 3 decimals.*

(c) Using each of the formulas, what is the expected U.S. wheat production in the year 2020? *Round to 2 decimals.*
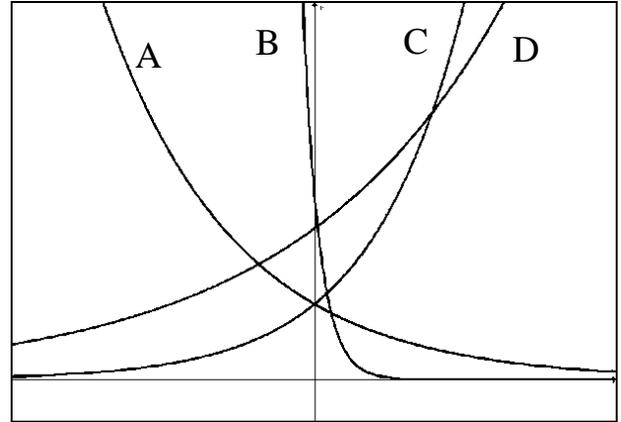
4. Find a formula for the exponential function.

(1, 4)

(3, 2.56)
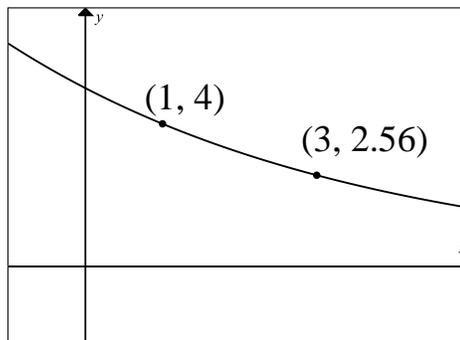
Section 4.3: Exponential Graphs

5. Assume the equations for A, B, C, and D can all be written in the form $y = ab^t$.

(a) Which function has the largest a value?

(b) Which two functions have the same a value?

(c) Which function has the smallest b value?

(d) Which function has the largest b value?

6. Let $f(x) = 5.4 + 3.1(1.6)^x$.

(a) As $x \to -\infty$, what happens to $f(x)$? In other words, what is $\lim_{x \to -\infty} f(x)$?

(b) Does $f(x)$ have a horizontal asymptote? If so, what is it?

Sections 4.4/4.5: The Number e

7. A 5-year certificate of deposit (CD) is opened with a $6,000 initial deposit at an annual rate of 2.9%, compounded monthly.

(a) How much money will be in the CD at the end of the 5 years? *Round to 2 decimals.*

(b) What is the effective rate for this CD? *Round the percent to 2 decimals.*

8. Suppose 2 mg of a drug is injected into a person's bloodstream. As the drug is metabolized, the quantity diminishes at the continuous rate of 4% per hour.

(a) Find a formula for $Q(t)$, the quantity of the drug remaining in the body after t hours, in the form $Q(t) = a \cdot e^{kt}$.

(b) Find a formula for $Q(t)$ in the form $Q(t) = a \cdot b^t$. *Round the decay factor to 4 decimals.*

(c) By what percent does the drug level decrease during any given hour? *Round to 2 decimals.*

(d) By what percent does the drug level decrease in 5 hours? *Round the percent to 2 decimals.*

Section 5.1: Logarithms

9. Solve for *x* exactly. Then, *round your answer to 2 decimals.*

(a) $\log(x+3) - \log(x-1) = 1$ (b) $6(1.2)^x = 5(1.7)^x$

10. Solve for *x*. *Give you answer in exact form.*

(a) $3e^{2x} + 5e^{2x} = 7$ (b) $\log(4x+5) \cdot \log(3x^2 + 1) = 0$

Section 5.2: Applications of Exponentials

11. If the price of a gallon of milk climbs at a continuous rate of 3% annually, how many years will it take its price to double? *Round to 2 decimals.*

12. Plutonium-238 has a half-life of 87.7 years. How long will it take a sample of plutonium-238 to decrease to 20% of its original mass? *Use the model $f(t) = ae^{kt}$ round your calculated value of k to 4 decimals. Round the final answer to a whole number.*

13. Suppose the amount of a radioactive substance is given by the function $f(t) = 20(0.645)^t$, where $f(t)$ is amount (in milligrams) of substance at time *t* years.
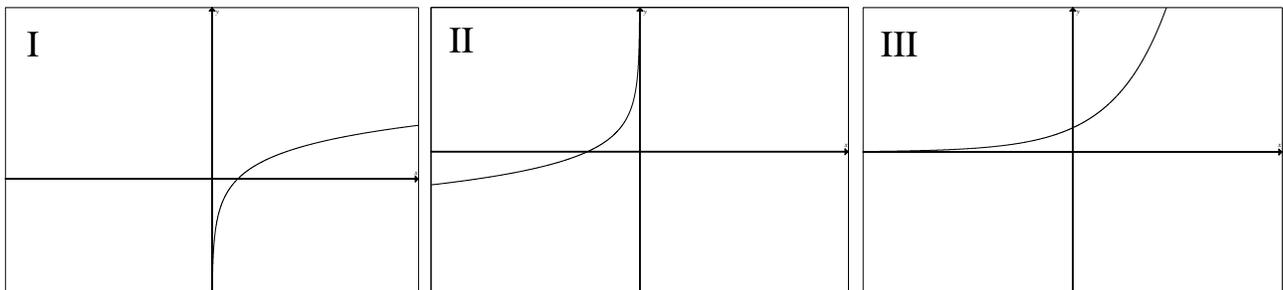
(a) Convert the exponential function to the form $f(t) = ae^{kt}$. *Round k to 4 decimals.*

(b) What is the continuous decay rate? What is the annual decay rate?

Section 5.3: Log Graphs

14. Graph the function $y = \ln(x+3)$. Identify any vertical asymptotes. State the domain and range of the function.

15. Match the statements (a)-(d) with the functions (I)-(III).

(a) $\lim\limits_{x \to -\infty} f(x) = 0$ (b) $\lim\limits_{x \to 0^-} f(x) = \infty$ (c) $\lim\limits_{x \to 0^+} f(x) = -\infty$

Sec. 6.1: Shifts and Reflections

16. The graphs of $f(x)$ and its translation $g(x)$ are shown. The point $(-1, 0)$ on $f(x)$ moves to the point $(3, -2)$ on $g(x)$. Give a possible formula for $g(x)$ in terms of $f(x)$.

17. A function $f(x)$ has domain $[-2,15)$ and range $[12,25)$. Find a possible formula for $h(x)$ in terms of $f(x)$, if the domain of $h(x)$ is $[-5,12)$ and the range is $(-25,-12]$.

18. The average rate of change of f over the interval $0 \le x \le 1$ is 4. Give the average rate of change of f over the interval $0 \le x \le 1$ for each transformation of f.

(a) $y = -2f(x)$ (b) $y = \dfrac{1}{3}f(x) + 2$

19. Determine (algebraically) whether the function is even, odd, or neither: $f(x) = \sqrt{x^2 + 1}$.

20. In the first quadrant, an *even* function f is decreasing and concave down. What can you say about the function's behavior in the second quadrant?

Sections 6.2/6.3: Vertical & Horizontal Stretches/Shrinks

21. The point $(-6, 7)$ lies on the graph of $g(x)$. What point lies on the graph of $y = 2g(x-3) + 4$?

22. Find a formula for the function g as a transformation of the function f.

23. Fill in all the blanks in the table for which you have sufficient information. If you do not have sufficient information to answer, write NA.

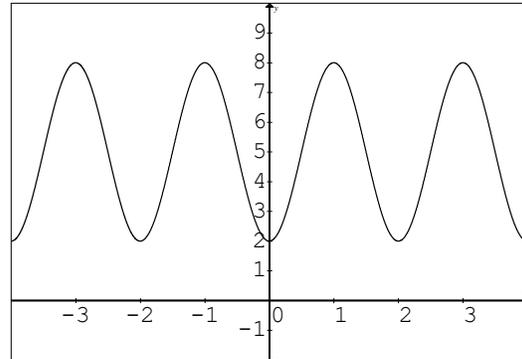| x	-2	-1	0	1	2
$f(x)$	-19	-3	1	-1	-3
$g(x) = -f(x) + 3$					
$h(x) = f\left(-\frac{1}{2}x\right)$					

Section 7.1: Periodicity

24. For the graph shown, estimate the period, midline, and the amplitude.

Period: _____

Midline: _____

Amplitude: _____

Section 7.2: Sine and Cosine

25. Without using a calculator, rank the following in order from smallest to largest.
$\cos 18°$, $\cos 12°$, $\cos 155°$, $\cos 212°$, $\cos 95°$, $\cos 85°$

26. Find angles between $0°$ and $360°$ that have the same
(a) cosine as $230°$ (b) sine as $305°$

27. Let θ be an angle in the first quadrant with $\cos\theta = a$ and $\sin\theta = b$. (See figure at right.) Evaluate each of the following in terms of a or b.

(a) $\cos(\theta + 180°)$ (b) $\sin(\theta + 180°)$
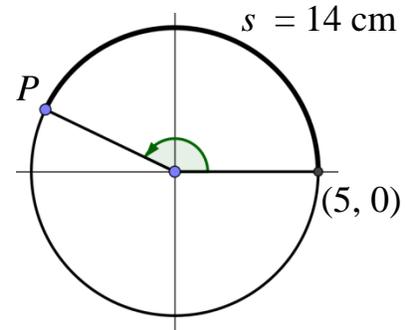
(c) $\cos(360° - \theta)$ (d) $\sin(360° - \theta)$
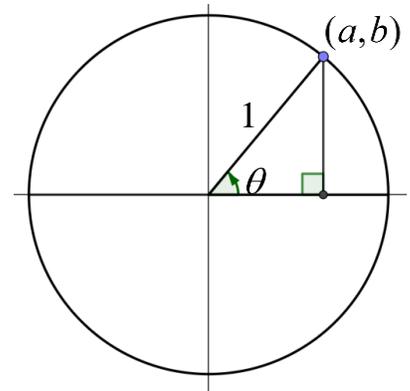
Section 7.3: Radians and Arc Length, Unit Circle

28. Find the **exact** coordinates of the point at an angle of $\dfrac{7\pi}{6}$ on a circle of radius 2 centered at the origin.

29. *P* is a point on a circle of radius 5 cm. The length of the arc from (5, 0) to *P* is 14 cm. What are the coordinates of point *P*? *Round to 2 decimals.*

$s = 14$ cm

P

(5, 0)

30. A weather satellite orbits the earth in a circular orbit 500 miles above the earth's surface. What is the radian measure of the angle (measured at the center of the earth) through which the satellite moves in traveling 600 miles along its orbit? (The radius of the earth is 3960 miles.) *Round to 4 decimals.*

Section 7.4: Sine and Cosine Graphs
31. Determine the amplitude, period, midline, and whether the graph has an *x*-axis reflection and/or a vertical shift. Then, write a possible equation for the graph.
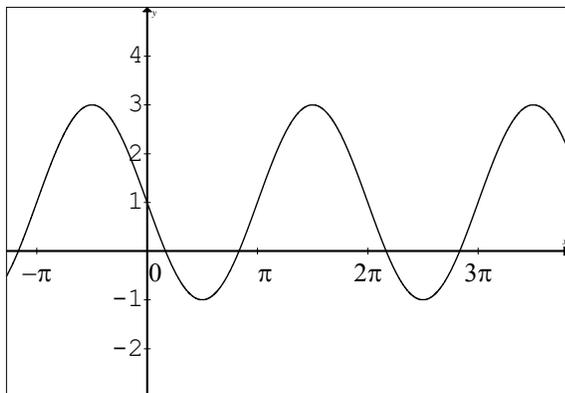
Amplitude: _____
x-axis reflection? *yes/no* (circle one)
Period:_____
Midline: $y =$ _____
Vertical shift:_____units *up/down* (circle one)
Equation: _____

32. For the function $f(x) = -4.2\sin(x) + 1.1$, find the amplitude, midline. In addition, find the domain and range (in interval notation) of *f*.

33. A mill has a water wheel that has a radius of 13 ft and the bottom of the wheel is 1 ft above the surface of the water. The wheel rotates counterclockwise, starting at the 3 o'clock position. Let *h* be the height (in feet) of the point *P* above the water as a function of the angle of θ radians. Find a formula for $h(\theta)$, and sketch a graph for $0 \le \theta \le 2\pi$.