MATH 1813, Exam 2(A), 100 points
March 27, 2019

> Only **one** calculator may be used on the exam. No TI-89, TI-92, TI-Nspire, or similar calculators may be used. Cell phone calculators, computers, laptops, and tablets are also prohibited. You **MUST show work** in the space provided to receive full credit.

1. (9 pts.) A town has an initial population of 3,500 people in the year 2015. Write a formula for the population, P, in t years after 2015 if the town:

(a) grows by 200 people per year.

(b) shrinks by 4% per year.

(c) grows at a continuous rate of 5% each year.

2. (8 pts. total) A typical cup of coffee contains about 100 mg of caffeine and every hour approximately 16% of the caffeine in the body is metabolized and eliminated.

(a) (6 pts.) Write a formula for the amount of caffeine C (in mg) in the body as a function of t, the number of hours since the coffee was consumed. **Show your work.**

(b) (2 pts.) What is $\lim_{t \to \infty} C(t)$? In other words, how much caffeine will the eventually be in the body, assuming no more coffee (or other caffeine) is consumed?

$\lim_{t \to \infty} C(t) = $ _____

3. (9 pts.) A student is growing bacteria in a test tube for an experiment. Two minutes after the start of the experiment, there are 84.5 million bacteria in the test tube. At five minutes, there are 93.7 million bacteria. Assuming the bacteria grow exponentially, find a formula for $P(t)$, the number of bacteria (in millions) t minutes after the start of the experiment. *Round the growth factor to 3 decimals and round the initial value to 1 decimal.* **Show your work.**

4. (6 pts.) Solve exactly for x. Then, provide your answer *rounded to 2 decimals.* **Show your work.**

$\log(3 \cdot 2^x) = 8$

5. (10 pts. total) A radioactive element has the decay model shown in the graph below.
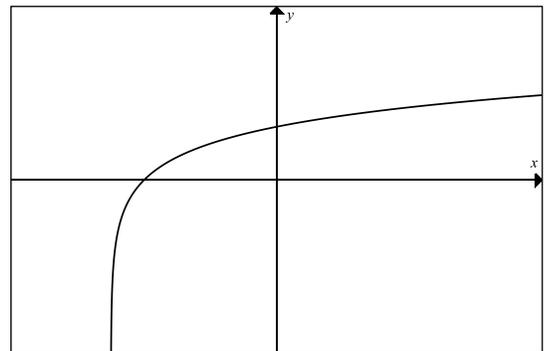
N = amount of element (mg)
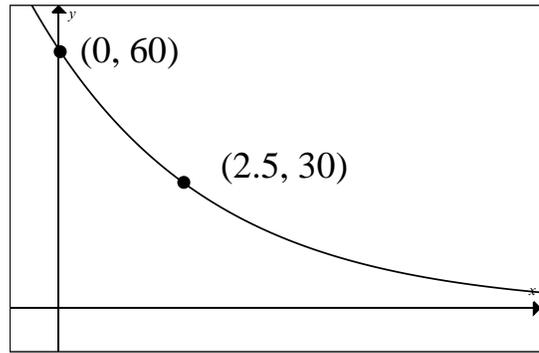
(a) (3 pts.) What is the half-life of this element? **Show your work.** *Include units.*

(0, 60)

(2.5, 30)

t = time (hours)

(b) (4 pts.) Find the continuous decay rate, k. *Round to 3 decimals.* **Show your work.**

(c) (3 pts.) Find a formula for N as a function of t, in the form $N = ae^{kt}$.

6. (10 pts. total) Let $f(x) = \log_b(x+5)$, where $b > 1$. The graph of f is represented at right.

(a) (2 pts.) What is the equation of the asymptote of the graph of f?

(b) (6 pts.) What are the domain and range of f?

Domain:_____

Range: _____

(c) (2 pts.) Complete the statement: $\lim\limits_{x \to -5^+} f(x) = $ _____.

7. (8 pts.) The table below shows values for a function g. Complete the table for $f(x) = g(-x) + 5$. **Show your work (or describe the transformations).**

x	-4	-2	2	4
$g(x)$	-43	-13	-1	-19

x	-4	-2	2	4
$f(x)$				

8. (6 pts.) The point $(-2, 7)$ lies on the graph of $y = f(x)$. Give the coordinates of one point on the graph of the following function. **Show your work by describing the transformations, in order.**
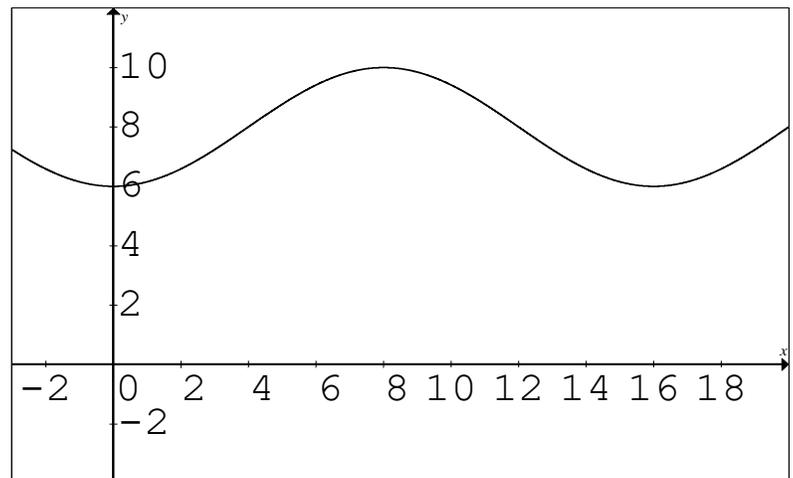
$y = f\left(\frac{1}{2}(x+3)\right) + 4$

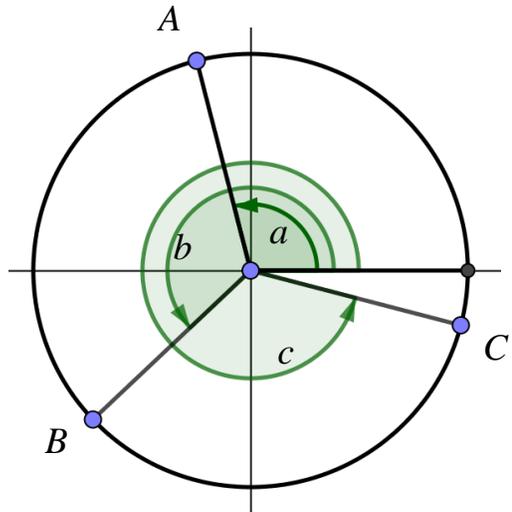9. (6 pts.) Estimate the period, midline and the amplitude of the following graph.

Period: _____

Midline: $y =$ _____

Amplitude:_____

10. (10 pts.) Points A, B, and C represent points on the unit circle that lie on the terminal sides of angles a, b, and c (respectively), in standard position.

(a) List the following in order from smallest to largest: $\sin(a)$, $\sin(b)$, $\sin(c)$

Explain your reasoning.

_____, _____, _____

(b) List the following in order from smallest to largest: $\cos(a)$, $\cos(b)$, $\cos(c)$.

Explain your reasoning.

_____, _____, _____

11. (10 pts.) Find angles between $0°$ and $360°$ that have the same:
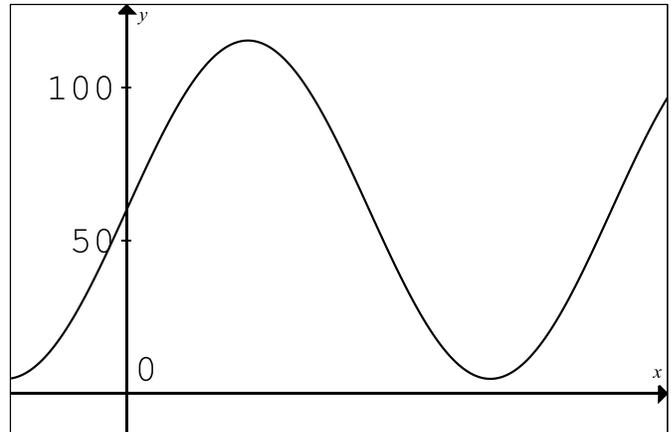
(a) Cosine as $190°$. **Show your work.**

(b) Sine as $190°$. **Show your work.**

12. (8 pts.) The height h above the ground (in meters) of a person on a Ferris wheel as a function of the angle θ measured from the 3 o'clock position is given by $h(\theta) = 55.3\sin(\theta) + 60.1$.

(a) What are the midline and amplitude of h? **Show your work.**

Midline: $y =$ _____

Amplitude: _____

(b) What are the highest and lowest points on the ride? **Show your work.** *Use correct units.*