INSTRUCTIONS: This exam is a **closed book exam.** You may **not** use your text, homework, or other aids except for a 3×5 inch notecard. You may use an allowable calculator, TI-83 or TI-84 to
- perform operations on real numbers,
- evaluate functions at specific values, and
- look at graphs and/or tables.

A TI-89, TI-Nspire, or a calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed.** Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that **correct answers using incorrect reasoning may not receive any credit.** Reasoning which will earn credit will use material covered in the course to date.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 8 problems on 11 pages. Make sure all problems and pages are present.

The exam is worth 67 points in total.

You have **60 minutes** to work starting from the signal to begin. Good luck!
### Exam 1 Grade by Problem Number

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<thead>
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<th>No.</th>
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<td><strong>Total</strong></td>
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### Current Course Grade by Category

<table>
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<th>Category</th>
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<td>WebAssign</td>
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<tr>
<td>Quiz/HW</td>
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<tr>
<td>Overall 6 Week Grade</td>
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1. (2 points each) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

(i) A candle has been burning at a constant rate of 1.25 inches per hour. The candle has been burning for 4 hours and is 5.5 inches tall. What was the length of the candle before it was lit?
   a. 5 inches
   b. 5.5 inches
   c. 6.75 inches
   d. 10.5 inches
   e. 0.5 inches

(ii) The graph below represents the height of water as a function of volume as water is poured into a container. Which container is represented by this graph?

(iii) Let \( f \) be a continuous function on the interval \([a, b]\). Which of the following is the most accurate interpretation of the average rate of change of \( f(x) \) with respect to \( x \) over the interval \([a, b]\)?
   a. The numerical mean of all the rates of change of \( f \) over the interval \([a, b]\).
   b. The constant rate of change of a linear function that has the same change in output values as the function \( f \) over the interval \([a, b]\).
   c. The average of the values \( f(b) \) and \( f(a) \).
   d. The change in the values of \( f(x) \) as \( x \) changes from \( a \) to \( b \).
   e. The numerical mean of all the values of \( f(x) \) over the interval \([a, b]\).
(iv) The following expression represents the derivative of what function?

\[
\lim_{\Delta x \to 0} \frac{2(x + \Delta x)^7 - 5(x + \Delta x) + 8 - (2x^7 - 5x + 8)}{\Delta x}
\]

a. \( f(x) = 2(x + \Delta x)^7 - 5(x + \Delta x) + 8 \)
b. \( f(x) = 2x^7 - 5x + 8 \)
c. \( f(x) = 2(x + \Delta x)^7 - 5(x + \Delta x) + 8 - (2x^7 - 5x + 8) \)
d. \( f(x) = 14x^6 - 5 \)
e. \( f(x) = \frac{2(x + \Delta x)^7 - 5(x + \Delta x) + 8 - (2x^7 - 5x + 8)}{\Delta x} \)

(v) Which of the following statements represents the exact meaning of \( f'(4.2) = 1.7 \).

I. The slope of the line tangent to the graph of \( y = f(x) \) at \( x = 4.2 \) is 1.7.
II. As \( x \) increases by 1 from \( x = 4.2 \), \( f(x) \) increases by 1.7.
III. The limiting value of the average rate of change of \( f \) over the interval \( [4.2, 4.2 + \Delta x] \) as \( \Delta x \) approaches zero is 1.7. (Note that \( \Delta x > 0 \).)

a. II only
b. I and II only
c. III only
d. I and III only
e. I, II, and III
2. (6 points) Use the limit definition of derivative to show that \( \frac{d}{dx} (x^2 - x) = 2x - 1 \).

You must use the definition of derivative to receive credit.
3. **(3 points each)** When running a marathon you heard the timer call out 12 minutes as you passed mile-marker 2.

(a) As you passed mile-marker 5 you heard the timer call out 33 minutes. What was your average speed from mile 2 to mile 5?

(b) If you passed mile marker 5 at 33 minutes, what average speed do you need to run for the remainder of the race to meet your goal of completing the 26.2-mile marathon in 175 minutes? (Round your answer to two decimal places.)
4. (a) (9 points) Answer the following questions based on the graph of $f$ below. Assume that all points of discontinuity and asymptotes can be observed from the graph. Asymptotes are indicated by dotted lines.

Give numeric values for each of the following. If the limit does not exist, write “$\infty$”, “$-\infty$”, or “DNE”, whichever is most appropriate.

\[
\lim_{x \to 3^-} f(x) = \quad f'(-4.5) = \quad \lim_{x \to -2} f(x) =
\]

\[
\lim_{\Delta x \to 0} \frac{f(-5+\Delta x)-f(-5)}{\Delta x} = \quad \lim_{x \to 0.5^+} f(x) = \quad \left. \frac{df}{dx} \right|_{x=3} =
\]

\[
\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \quad \lim_{x \to 5} f(x) = \quad \lim_{x \to -3^+} f(x) =
\]

(b) (1 point) Identify an $x$-value in the interval $[-10, 10]$ where $f$ is continuous but not differentiable.
5. (3 points each) Consider the function $f$ defined by

$$f(x) = \begin{cases} 
2x^2 - 7x - 6, & x \leq 1 \\
-5 - 8x, & x > 1 
\end{cases}$$

(a) Compute $\lim_{x \to 1^-} f(x)$.

(b) Compute $\lim_{x \to 1^+} f(x)$.

(c) Does $\lim_{x \to 1} f(x)$ exist? Justify your response.

(d) Is $f$ continuous at $x = 1$? Justify your response.
6. (3 points each) Compute the following derivatives. **Do not simplify** after taking each derivative.

(a) Let $f(x) = 3x^4 - x^2 + 11x$. Find $f'(x)$.

(b) Let $y = x - \frac{5}{x^5}$. Find $\frac{dy}{dx}$.

(c) Let $g(x) = e^{\sqrt{x}}$. Find $g'(x)$.

(d) Let $y = \frac{x^8}{x + 9}$. Find $\frac{dy}{dx}$. 
7. (2 points each) The graph of the function $y = f(x)$ is given below. Determine values for $a$, $b$, and $c$ that satisfy the respective inequality or equality. (Note that there are multiple values for $a$, $b$, and $c$ that satisfy these (in)equalities.)

(a) Determine a single numerical value for the constant $a$ that makes the following inequality true:

$$\frac{f(2) - f(a)}{2 - a} < f'(2)$$

$$a =$$

(b) Determine a single numerical value for the constant $b$ that makes the following inequality true:

$$-1 < \frac{f(b + 0.5) - f(b)}{0.5} < 0$$

$$b =$$

(c) Determine a single numerical value for the constant $c$ that makes the following equality true:

$$\lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = 0$$

$$c =$$
8. (5 points) In the space provided to the right of each graph, write the letter(s) of the expression(s) which are valid for the graph. Each graph will have at least one matching expression and could have multiple matching expressions. Note that not all expressions will be used and you can use each expression more than once. For each graph, assume that the obvious patterns continue.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Graph</th>
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<tbody>
<tr>
<td>(A) ( \lim_{x \to a} f(x) ) exists</td>
<td>Jump Discontinuity</td>
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<tr>
<td>(B) ( f(a) ) exists</td>
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<tr>
<td>(C) ( \lim_{x \to a} f(x) ) exists but ( \lim_{x \to a} f(x) \neq f(a) )</td>
<td>Removable Discontinuity</td>
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<tr>
<td>(D) ( \lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x) )</td>
<td>Vertical Asymptote</td>
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<td>(E) ( \lim_{x \to a} f(x) = f(a) )</td>
<td>Continuous at ( x = a )</td>
</tr>
<tr>
<td>(F) ( \lim_{x \to \infty} f(x) ) exists (as a number)</td>
<td>Horizontal Asymptote</td>
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<tr>
<td>(G) ( \lim_{x \to -\infty} f(x) ) exists (as a number)</td>
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<td>(H) ( \lim_{x \to a^-} f(x) = \infty )</td>
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<tr>
<td>(I) ( \lim_{x \to a} f(x) = \infty )</td>
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