$A=(1,2,0)\quad B=(3,5,1)\quad C=(-1,4,5)$

$\vec{BA}=\langle 1-3,\ 2-5,\ 0-1\rangle=\langle -2,-3,-1\rangle$

$\vec{BC}=\langle -1-3,\ 4-5,\ 5-1\rangle=\langle -4,-1,+4\rangle$

proj. of $\underset{u}{BA}$ along $\underset{v}{BC}$ $\quad \left(\dfrac{u\cdot v}{v\cdot v}\right)v$

$$\frac{\langle -2,-3,-1\rangle\cdot\langle -4,-1,4\rangle}{\langle -4,1,4\rangle\cdot\langle -4,1,4\rangle}=\frac{8+3-4}{16+1+16}=\frac{7}{33}$$

$$\frac{7}{33}\langle -4,-1,4\rangle=\left\langle \frac{-28}{33},\frac{-7}{33},\frac{28}{33}\right\rangle$$

$\angle ABC\quad \theta=\cos^{-1}\left(\dfrac{u\cdot v}{\|u\|\|v\|}\right)$

$u=BA=\langle -2,-3,-1\rangle$

$\|u\|=\sqrt{4+9+1}=\sqrt{14}$

$v=BC=\langle -4,-1,4\rangle$

$\|v\|=\sqrt{16+1+16}=\sqrt{33}$

$$\theta=\cos^{-1}\left(\frac{7}{\sqrt{14}\sqrt{33}}\right)$$

$\approx 1.239\text{rad}$

$\underset{u}{\langle -2,-3,-1\rangle}\quad \underset{v}{\langle -4,-1,4\rangle}$

$$\begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & -3 & -1\\ -4 & -1 & 4\end{bmatrix}$$

$\text{Area}(T)=\dfrac{\|u\times v\|}{2}$

$u\times v=\langle -12-1,\ -(-8-4),\ 2-12\rangle$

$=\langle -13,12,-10\rangle$

$$\|u\times v\|=\frac{\sqrt{13^2+12^2+10^2}}{2}$$

$f(x,y) = e^{xy - x^2} \qquad P = (1,2)$

$\nabla f = \langle e^{xy-x^2}(y-2x),\ e^{xy-x^2}(x) \rangle$

$\nabla f_P = \langle 0, e \rangle \quad P = (1,2)$

$z = f(1,2) + f_x(1,2)(x-1) + f_y(1,2)(y-2)$

$z = e + \cancel{0(x-1)} + e(y-2)$

$z = e + e(y-2)$

$r(t) = \langle \sin(t), \cos(t), 2t \rangle$

$r'(t) = \langle \cos(t), -\sin(t), 2 \rangle$

$s = \|r'(t)\| = \sqrt{\cos^2 t + \sin^2(t) + 4} = \underline{\sqrt{5}}$

Arc Length from $0 \le t \le \frac{\pi}{2}$

$l = \int_0^{\frac{\pi}{2}} \sqrt{5}\,dt = \sqrt{5}\,t \Big|_0^{\pi/2} = \frac{\sqrt{5}\,\pi}{2}$

$\int F \cdot dr \quad F = \langle xy, 0, x \rangle \quad \underline{0 \le t \le \frac{\pi}{2}}$

$F(r(t)) = \langle \sin t \cos t, 0, \sin t \rangle$

$\int F \cdot dr = \int_a^b F(r(t)) \cdot r'(t)\,dt$

$\int_0^{\pi/2} \langle \sin t \cos t, 0, \sin t \rangle \cdot \langle \cos t, -\sin t, 2 \rangle\,dt$

$\int_0^{\pi/2} \sin t \cos^2 t + 0 + 2\sin t\,dt$

$\frac{-7}{3}$?

$f(x,y) = 4x - 3x^3 - 2xy^2$

$f_x = 4 - 9x^2 - 2y^2 \qquad f_y = -4xy$

$4xy = 0$

$x = 0 \qquad y = 0$

$4 - 9x^2 - 2y^2 = 0$

$\underline{x=0}$

$4 - 2y^2 = 0$

$4 = 2y^2$

$y^2 = 2$

$y = \pm\sqrt{2}$

$y = 0$

$4 - 9x^2 = 0$

$\frac{4 = 9x^2}{9}$

$\frac{4}{9} = x^2$

$x = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$

$$\boxed{\begin{matrix}(0,-\sqrt{2})\\(0,\sqrt{2})\\(-\frac{2}{3},0)\\(\frac{2}{3},0)\end{matrix}}$$

Crit. pts.

$f_x = 4 - 9x^2 - 2y \qquad f_y = -4xy$

$f_{xx} = -18x \qquad f_{yy} = -4x$

$f_{xy} = -4y = f_{yx}$

$D = \det\begin{bmatrix} -18x & -4y \\ -4y & -4x \end{bmatrix} = 72x^2 - 16y^2$

$0 - 16(2)$

$72(\frac{2}{3})^2 - 0$

$-18x$

$-18(-\frac{2}{3}) = +$

$-18(\frac{2}{3}) = -$

$(-\frac{2}{3}, 0)$ − min

$(\frac{2}{3}, 0)$ − max

crit pt	D
saddlepoint $(0,-\sqrt{2})$	$D < 0$
$(0,\sqrt{2})$	$D < 0$
$(-\frac{2}{3},0)$	$D > 0$
$(\frac{2}{3},0)$	$D > 0$

b

$\underline{z = \sqrt{x^2+y^2}}$ $\qquad$ $x^2+y^2+z^2=1$ $\qquad$ $\rho^2 \sin\phi$

$\rho^2 = 1$

$\rho\cos\phi = \sqrt{\rho^2\cos^2\theta\sin^2\phi + \rho^2\sin^2\theta\sin^2\phi}$

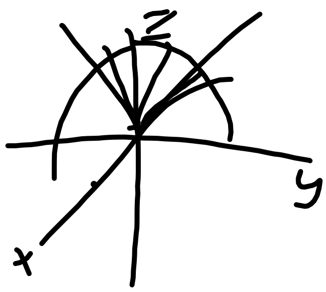
$= \sqrt{\rho^2\sin^2\phi\left(\cos^2\theta + \sin^2\theta\right)}$

$\rho\cos\phi = \sqrt{\rho^2\sin^2\phi}$

$= \rho\sin\phi$

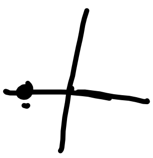
$\cos\phi = \sin\phi$

$0 \le \phi \le \frac{\pi}{4}$



$$\int_0^1\int_0^{2\pi}\int_0^{\frac{\pi}{4}} \rho^2\sin\phi \, d\phi\, d\theta\, d\rho$$

$$\lim_{(x,y)\to(-1,0)} \frac{x^2-y^2}{x^2+y^2} \quad \lim_{(r,\theta)\to(1,\pi)} \frac{r^2\cos^2\theta - r^2\sin^2\theta}{r^2}$$



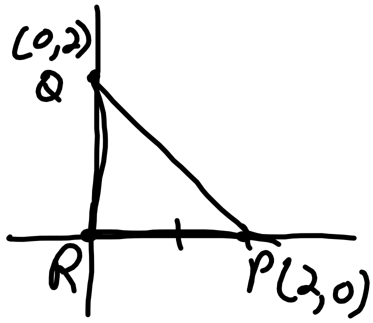
$(1,\pi)$

$$\lim_{(r,\theta)\to(1,\pi)} \frac{\cancel{r^2}(\cos^2\theta-\sin^2\theta)}{\cancel{r^2}}$$

$$=\frac{(-1)^2-0^2}{1}=1$$

$$\oint_{\partial D} F_1\,dx + F_2\,dy = \iint_D \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\,dA$$

$$\underbrace{\oint_C \overset{F_1}{\underline{xy^2}}\,dx + \overset{F_2}{\underline{x}}\,dy}_{C = \text{unit circle}} = \underbrace{\iint_C 1 - 2xy\,dA}_{\int_0^1\int_0^{2\pi}(1-2r^2\cos\theta\sin\theta)\,r\,d\theta\,dr}$$



$$c(t) = (\overset{x}{2-t}, \overset{y}{t})$$

$P \to Q$

PQ

$t \in [0,2]$

$$F = \langle xy, x \rangle$$

$$\int_{PQ} \underline{xy}\,dx + \underline{x}\,dy = \int_{PQ} F \cdot dr$$

$$\int_0^2 F(c(t)) \cdot c'(t)\,dt \qquad c'(t) = \langle -1, 1 \rangle$$

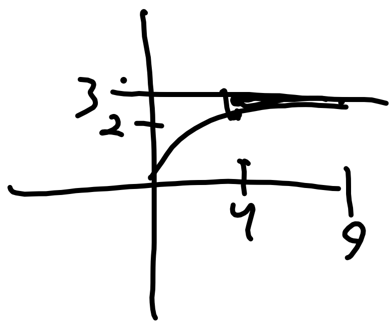
$$F(c(t)) = \langle (2-t)t, 2-t \rangle$$

$$\int_0^2 -1(2-t)t + 2-t\,dt$$

$$\int_0^2 t^2 - 2t + 2 - t\,dt$$

$$\int_0^2 t^2 - 3t + 2\,dt$$

$$\int_4^9 \int_{\sqrt{x}}^3 f(x,y)\,dy\,dx$$



$y=\sqrt{x}$

$x=y^2$

$$\int_2^3 \int_4^{y^2} f(x,y)\,dx\,dy$$