INSTRUCTIONS: This exam is a closed book exam. You may not use your text, homework, or other aids except for a 3 × 5-inch notecard. You may use an allowable calculator, TI-83 or TI-84 to
• perform operations on real numbers,
• evaluate functions at specific values, and
• look at graphs and/or tables.
A TI-89, TI-Nspire, or a calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are not allowed. Unless otherwise stated, you must show all of your work including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that correct answers using incorrect reasoning may not receive any credit. This comprehensive exam assesses your understanding of material covered throughout the entire course.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 10 problems on 13 pages. Make sure all problems and pages are present.

The exam is worth 100 points in total.

You have 1 hour and 50 minutes to work starting from the signal to begin. Good luck!
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1. (2 points each) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

(i) If \( f \) is a differentiable function that has a local maximum at \( x = c \), then which of the following is true?

a. \( f'(c) < 0 \)

b. \( f'(c) > 0 \)

c. \( f'(c) = 0 \)

d. \( f'(c) > f'(x) \) for all \( x \) in the domain of \( f' \)

e. \( f'(c) < f'(x) \) for all \( x \) in the domain of \( f' \)

(ii) The derivative function \( y = f'(x) \) is graphed below.

![Graph of f'(x)](image)

Based on this graph, the function \( y = f(x) \) is decreasing on the interval(s)?

a. \((-\infty, -1)\)

b. \((-1, 1) \) and \((4, \infty)\)

c. \((0, 2.75)\)

d. \((-\infty, -1)\) and \((1, 4)\)

e. \((4, \infty)\)

(iii) Suppose \( f(x) > 0 \) and \( f'(x) < 0 \) for \( 2 \leq x \leq 4 \). Which of the following approximations of \( \int_{2}^{4} f(x) \, dx \) is the largest?

a. \( R_4 \)

b. \( L_4 \)

c. \( M_4 \)

d. They are all equal.

e. There is not enough information provided to determine which approximation is largest.
(iv) Consider the graph of \( f(x) = x \sin(x) \) on the domain \([-4, 3]\) below. How many values of \( c \) in \((-4, 3)\) appear to satisfy the Mean Value Theorem equation 

\[
f'(c) = \frac{f(b) - f(a)}{b - a}
\]

where \( a = -4 \) and \( b = 3 \).

a. None  
b. One  
c. Two  
d. Three  
e. Four or more

(v) The graph of the function \( f(x) = \sin(2x) \) is shown below.

The value of the expression \( \int_{0.2}^{0.8} \sin(2x) \, dx \) is equal to the area of the shaded region in the graph above. The value of which of the following expressions is also equal to the area of the shaded region?

I. \( \frac{1}{2} \int_{0.4}^{1.6} \sin(\theta) \, d\theta \)

II. \( \Delta x \sum_{k=1}^{N} \sin(2(0.2 + k\Delta x)) \)

III. \( \frac{1}{2} (-\cos(1.6) + \cos(0.4)) \)

a. I only  
b. I and III only  
c. III only  
d. II only  
e. I, II, and III
2(a) (9 points) Answer the following questions based on the graph of $y = f(x)$ below. Assume that all critical points, points of discontinuity, points of inflection, and the end behavior of $f$ can be observed from the graph below. Asymptotes are indicated by dotted lines.

Give numeric values for each of the following. Write “DNE” if the value does not exist and “$\infty$” or “$-\infty$” as appropriate.

\[
\lim_{x \to -4^+} f(x) = \quad f'(8) = \quad \lim_{x \to 5^-} f(x) = \\
\int_{-8}^{-6} f(x) \, dx = \quad \frac{d^2 f}{dx^2} \bigg|_{x = -5} = \quad \lim_{\Delta x \to 0} \frac{f(4 + \Delta x) - f(4)}{\Delta x} = \\
\frac{f(7.3) - f(6)}{7.3 - 6} = \quad \lim_{x \to -\infty} f(x) = \quad f''(0) =
\]

2(b) (1 point) Identify a value of $x$ for which $f$ is continuous but not differentiable.
3. (6 points) A continuous function \( y = f(x) \) defined on the interval \(-1 \leq x \leq 2\) has the following properties:

\[
\begin{align*}
    f(0) &= 3 \\
    f'(0) &= 0 \text{ and } f''(-0.8) = f''(1) = 0 \\
    f'(x) &> 0 \text{ on the interval } -1 \leq x < 0 \\
    f'(x) &< 0 \text{ on the interval } 0 < x \leq 2 \\
    f''(x) &> 0 \text{ on the intervals } -1 \leq x < -0.8 \text{ and } 1 < x \leq 2 \\
    f''(x) &< 0 \text{ on the intervals } -0.8 < x < 1
\end{align*}
\]

Sketch a graph of the function on the axes below.
4. (10 points) The graph of the function $f$ is shown below. Use this graph for answering parts (b) and (c).

(a) (4 points) Suppose $f$ is a function and suppose $G(x) = x \cdot f(x^2)$. Refer to the graph of $f$ above to compute $G'(2)$.

(b) (2 points each) Let $A(x) = \int_0^x f(t) \, dt$. Refer to the graph of $f$ above to evaluate the following:

i. $A(2) =$

ii. $A'(3) =$

iii. $A''(1) =$
5. (4 points each) Compute the following derivatives. Do not simplify.

(a) Let $y = \frac{\sin(x)}{4x}$. Find $\frac{dy}{dx}$.

(b) Let $f(t) = \sqrt{t^2 + 3}$. Find $f'(t)$.

(c) Let $g(x) = \tan^{-1}(\cos(x))$. Find $g'(x)$.

(d) Let $h(\theta) = \sec(\ln(\theta))$. Find $\frac{dh}{d\theta}$.
6. (6 points) Consider the curve defined by \( y^3 - \frac{x}{y} = 4 \). Use implicit differentiation to determine \( \frac{dy}{dx} \). (You must solve for \( \frac{dy}{dx} \) in terms of \( x \) and \( y \).)

7. (6 points) The volume of a sphere is given by the equation \( V = \frac{4}{3} \pi r^3 \), where \( r \) is the radius of the sphere (in meters). Suppose \( r \) is increasing at a rate of 3 meters per second. Determine the rate at which the volume is increasing when \( r = 2 \) meters.
8. (10 points) The top and bottom of a cylinder will be constructed from square pieces of cardboard (with the excess material thrown away). The side (i.e., lateral surface) of the cylinder will be made of a rectangular piece of cardboard. Altogether, 2,400 square inches of cardboard will be used (including the wasted amount). What is the radius of the cylinder of largest volume that can be constructed in this way? Justify your response.

(Recall that the volume of a cylinder is given by \( V = \pi r^2 h \).)
9. (4 points each) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

(a) \( \int 3 \sec^2(-8x) \, dx = \)

(b) \( \int_1^4 \frac{2}{t^2} \, dt = \)

(c) \( \frac{d}{dx} \int_\pi^x \frac{\sin(\theta)}{\theta} \, d\theta = \)

(d) \( \int \frac{\sec^2(x)}{\tan(x)} \, dx = \)
10. (6 points) Let $R$ be the region bounded by the graphs of $y = 1 - x^2$ and $y = 1 - x$ over the interval $[0, 1]$. (These graphs are shown below.)

(a) Find the area of the region $R$.

(b) (4 points) Suppose the region $R$ is rotated around the $x$-axis. Completely set up the integral that represents the volume of the resulting solid but do not evaluate this integral.
BASIC FORMULAS

\[
\frac{d}{dx}x^n = nx^{n-1}
\]

\[
\frac{d}{dx}e^x = e^x
\]

\[
\frac{d}{dx}\ln(x) = \frac{1}{x}
\]

\[
\frac{d}{dx}\log_a(x) = \frac{1}{x \ln(a)}
\]

\[
\frac{d}{dx}a^x = a^x \ln(a)
\]

\[
\frac{d}{dx}\sin(x) = \cos(x)
\]

\[
\frac{d}{dx}\cos(x) = -\sin(x)
\]

\[
\frac{d}{dx}\tan(x) = \sec^2(x)
\]

\[
\frac{d}{dx}\cot(x) = -\csc^2(x)
\]

\[
\frac{d}{dx}\sec(x) = \sec(x) \tan(x)
\]

\[
\frac{d}{dx}\csc(x) = -\csc(x) \cot(x)
\]

\[
\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}
\]

\[
\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}
\]

\[
\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}
\]

\[
\frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}
\]

\[
\int u^n \,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1
\]

\[
\int \frac{du}{u} = \ln|u| + C
\]

\[
\int e^u \,du = e^u + C
\]

\[
\int a^u \,du = \frac{a^u}{\ln a} + C
\]

\[
\int \sin(u) \,du = -\cos(u) + C
\]

\[
\int \cos(u) \,du = \sin(u) + C
\]

\[
\int \sec^2(u) \,du = \tan(u) + C
\]

\[
\int \csc^2(u) \,du = -\cot(u) + C
\]

\[
\int \sec(u) \tan(u) \,du = \sec(u) + C
\]

\[
\int \csc(u) \cot(u) \,du = -\csc(u) + C
\]

\[
\int \tan(u) \,du = \ln|\sec u| + C
\]

\[
\int \cot(u) \,du = \ln|\sin u| + C
\]

\[
\int \sec(u) \,du = \ln|\sec u + \tan u| + C
\]

\[
\int \csc(u) \,du = \ln|\csc u + \cot u| + C
\]

\[
\int \frac{du}{\sqrt{a^2-u^2}} = \frac{1}{a} \sin^{-1}\left(\frac{u}{a}\right) + C
\]

\[
\int \frac{du}{a^2+u^2} = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C
\]

\[
\int f(u(x)) \cdot u'(x) \,dx = \int f(u) \,du
\]

\[
L_n = \Delta x \sum_{k=1}^{n} f(a + (k - 1)\Delta x)
\]

\[
R_n = \Delta x \sum_{k=1}^{n} f(a + k\Delta x)
\]

\[
M_n = \Delta x \sum_{k=1}^{n} f\left(a + \frac{2k - 1}{2} \Delta x\right)
\]