1. (*2 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (i) Mara is traveling down Interstate 35 at a constant velocity. For every increase of ten minutes in the number of minutes x since she passed mile marker 174, her distance y from Downtown Oklahoma City decreases by seven miles. Which of the following statements is definitely true?

   a. $\Delta y = -0.7\Delta x$
   b. $y = -0.7x$
   c. $x \approx -1.43y$
   d. Statements (a) and (b) are definitely true
   e. Statements (a), (b), and (c) are definitely true

   (ii) Suppose that $y = f(x)$ is any function such that $f(2)$ and $f(6)$ are both defined. Which of the following statements are true?

   I. It is always true that $f(6) = 3 \cdot f(2)$
   II. It is always true that $f(4)$ is also defined, because $f(4) = f(6) - f(2)$
   III. It is always true that $f(3)$ is also defined, because $f(3) = \dfrac{f(6)}{f(2)}$

   a. Only statements I and II are always true
   b. Only statements II and III are always true
   c. Only statements I and III are always true
   d. All of these statements are always true
   e. None of these statements are always true

   (iii) Consider the function $f(x) = x - \sqrt{x}$. Which of the following functions gives the average rate of change of $f(x)$ with respect to x on the interval from $x = 9$ to $x = 9 + h$?

   a. $g(h) = \dfrac{9 + h - \sqrt{9 + h}}{h}$
   b. $g(h) = \dfrac{9 + h - 9}{9 + h - \sqrt{9 + h} - 6}$
   c. $g(h) = \dfrac{9 + h - \sqrt{9 + h} - \left(9 - \sqrt{9}\right)}{h}$
   d. $g(h) = \dfrac{3}{h}$
   e. $g(h) = 1 + \dfrac{1}{2\sqrt{x}}$

(iv) Suppose that a function $y = f(x)$ has a jump discontinuity at an input value $x = a$. Which of the following statements must be true if the function f is continuous at all other input values?

a. $f(a)$ must be undefined
b. $f(a)$ must be defined
c. $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exist and are not equal
d. $\lim_{x\to a^-} f(x) = f(a)$ or $\lim_{x\to a^+} f(x) = f(a)$
e. $\lim_{x\to a^-} f(x) = f(a)$ and $\lim_{x\to a^+} f(x) = f(a)$

(v) Which function below provides the average rate of change for the function $f(t) = 3 - 2\sqrt{t}$ on the input interval from t to $t + 27$?

a. $g(t) = \dfrac{3 - 2\sqrt{t+27} - \left(3 - 2\sqrt{t}\right)}{27}$

b. $g(t) = -\dfrac{6 + 2\sqrt{27+t}}{t}$

c. $g(t) = \dfrac{6 - 2\sqrt{t+27} - 2\sqrt{t}}{27}$

d. $g(t) = \dfrac{6 - 2\sqrt{27+t}}{t}$

e. $g(t) = -\dfrac{3 - 2\sqrt{27+t}}{3 - 2\sqrt{27}}$

(vi) At what input values does the function f below have a removable discontinuity?

$$f(x) = \frac{x^2 - 1}{x^2 + 3x + 2}$$

a. There are no removable discontinuities
b. Only at $x = 1$
c. Only at $x = -1$
d. Only at $x = -1$ and $x = -2$
e. Only at $x = -1$, $x = 1$, and $x = 2$

(vii) If $\lim_{x\to a} f(x) = L$, which of the following must be true?

I. $f(a) = L$
II. $\lim_{x\to a^-} f(x) = L$
III. $\lim_{x\to a^+} f(x) = L$

a. I only
b. I and II
c. I and III
d. II and III
e. I, II, and III

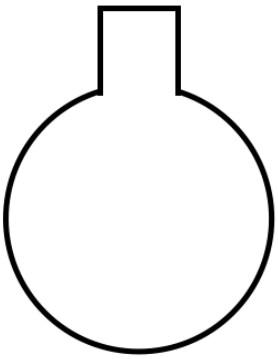
2. (*3 points each*) Suppose a bottle is being filled with water. Let $h(V)$ represent the height of the water in the vase (in inches) and let V represent the volume of water in the bottle (in fluid ounces).

   (a) Explain what the expression $h(56) - h(23)$ represents in the context of this situation.

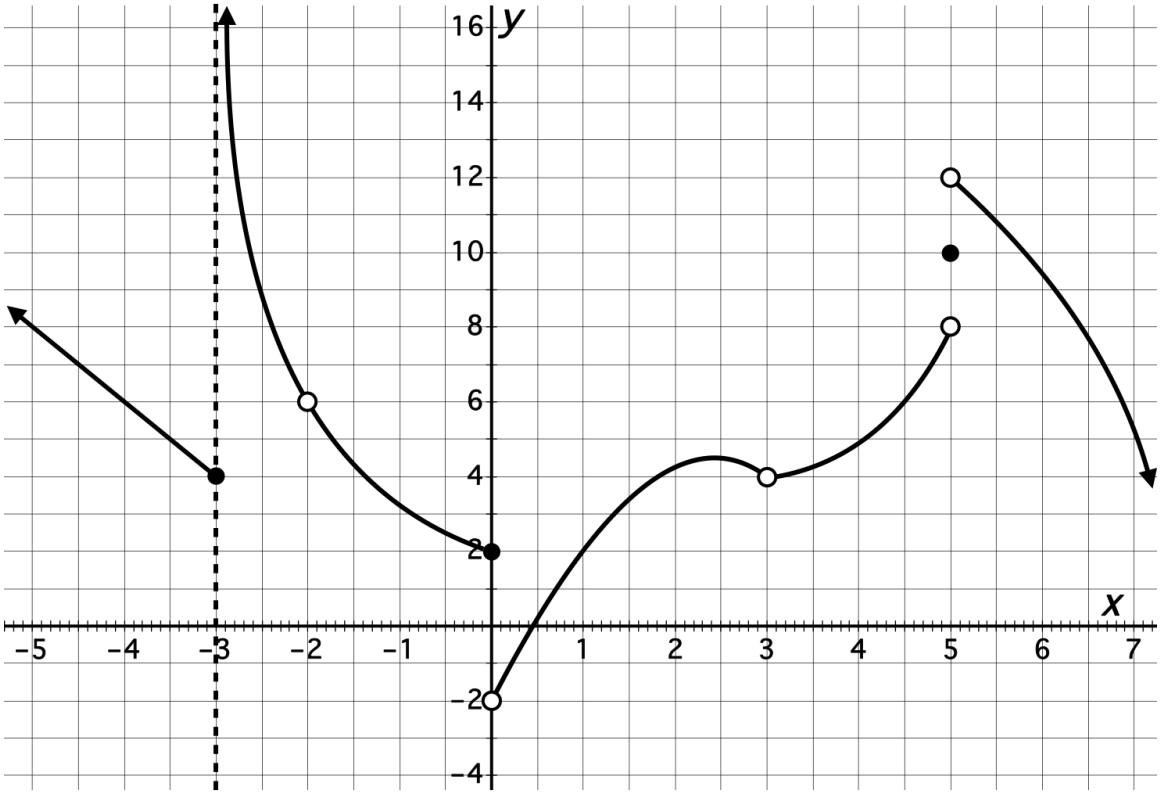
   (b) Explain what the solution to the equation $h(V) = 4.7$ represents in the context of this situation.

   (c) Explain what the equation $h^{-1}(7.5) = 60.1$ means in the context of this situation.

   (d) Suppose the image below illustrates the bottle being filled with water. On the axes provided, sketch a graph of the height of water in the bottle (in inches) as a function of the volume of water in the bottle (in fluid ounces).



3. (a) Answer the following questions based on the graph of the function f below. Dashed lines indicate asymptotes. Assume that obvious patterns continue. (*1 point each*)



(*1 point each*) Give numeric values for each of the following. If the limit does not exist, write "∞", "$-\infty$", or "DNE", whichever is most appropriate.

$\lim_{x\to -4} f(x) =$ $\qquad$ $f(-3) =$ $\qquad$ $\lim_{x\to -3^-} f(x) =$

$\lim_{x\to 5^+} f(x) =$ $\qquad$ $\lim_{x\to -3^+} f(x) =$ $\qquad$ $\lim_{x\to 0} f(x) =$

$f(5) - f(0) =$ $\qquad$ $\lim_{\Delta x\to 0} \frac{f(-4+\Delta x)-f(-4)}{\Delta x} =$ $\qquad$ $\lim_{x\to 3} f(x) =$

(b) (*2 points*) Identify all x-values in the interval $[-5, 7]$ where f is not continuous.

(c) (*2 points*) What is the domain of f?

4. (*4 points each*) Let f be a function defined by

$$f(x) = \begin{cases} \dfrac{3}{x-1}, & x \leq 0 \\ \dfrac{1}{x^2-25}, & x > 0 \end{cases}$$

   (a) Is the function f differentiable at $x = 0$? Explain.

   (b) Is the function f continuous at $x = 0$? Use the definition of continuity at a point to justify your response.

5. (*8 points*) Use the definition of derivative to show that $\dfrac{d}{dx}\left(3x^2 - 4\right) = 6x$. You **must** use the **definition of derivative** to receive any credit.

6. (*4 points each*) Let $f(x)$ be a continuous function with the following known values.

x	-3	-2	-1	0	1	2	3
$f(x)$	7	-0.5	-1	1	2	3	-4

(a) Does $f(x) = -3.1$ have a solution? Justify your response.

(b) What is the minimum number of zeros that $f(x)$ must have? Justify your response.

7. Compute the following derivatives.

   (a) $f(x) = 5x^7 - 2x^3 + e^x$. Find $f'(x)$.

   (b) $y = 4\sqrt{x} - \pi x$. Find $\dfrac{dy}{dx}$.

   (c) $g(t) = t(t^2 - 9t)$. Find $g'(t)$.