

**INSTRUCTIONS:** This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a  $3 \times 5$  inch notecard. You may use an allowable calculator, **TI-83 or TI-84** to

- perform operations on real numbers,
- evaluate functions at specific values, and
- look at graphs and/or tables.

A TI-89, TI-Nspire, or a calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that **correct answers using incorrect reasoning may not receive any credit**. Reasoning which will earn credit will use material covered in the course to date.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 7 problems on 10 pages. Make sure all problems and pages are present.

The exam is worth 66 points in total.

You have **60 minutes** to work starting from the signal to begin. Good luck!

**Exam 1 Grade by  
Problem Number**

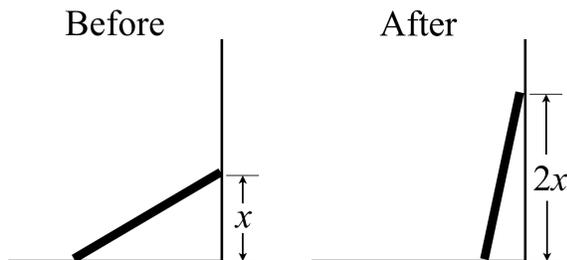
No.	Out of	Pts.
1	10	
2	12	
3	11	
4	6	
5	9	
6	12	
7	6	
Total	66	

**Current Course Grade by Category**

Category	Out of	Current
Exam 1	100%	
WebAssign	100%	
Quiz/HW	100%	
Overall 6 Week Grade	100%	

1. (2 points each) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

- (i) Suppose a wooden board is leaning against a wall. Now suppose that the slant of the board is adjusted so that it reaches twice as high on the wall (see the image below).



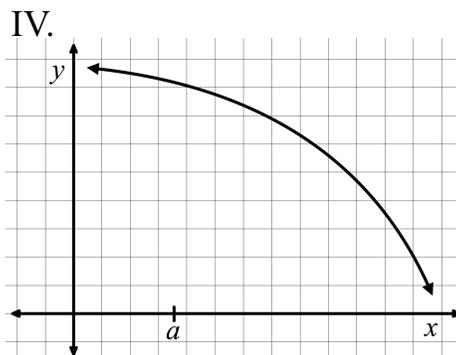
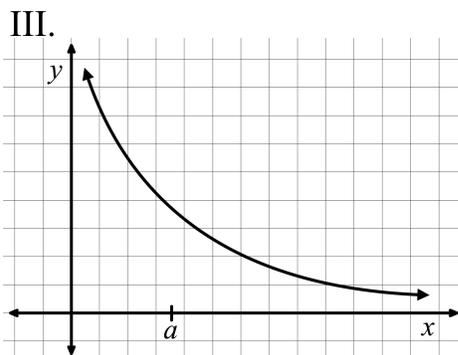
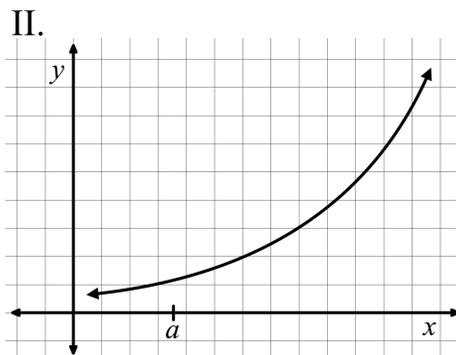
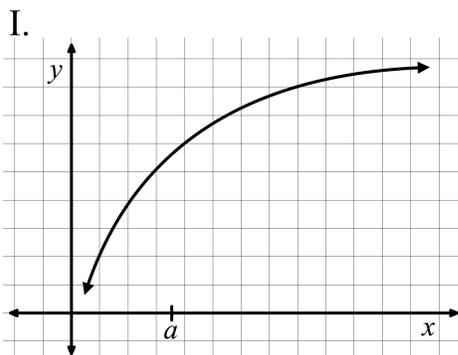
The slope of the board is:

- More than twice what it was before
  - Exactly twice what it was before
  - Less than twice what it was before
  - The same as what it was before
  - There is not enough information to answer this question
- (ii) An object is moving forward along a straight line. The distance the object has traveled (in meters) from its starting position  $t$  seconds after it started moving is given by  $s(t) = 2\sqrt{t}$ . What is the average velocity of the object over the interval from  $t = 4$  to  $t = 9$ .
- 5
  - 1
  - $\frac{2}{5}$
  - 2
  - $\frac{5}{2}$
- (iii) The temperature  $T$  (in degrees Fahrenheit) of a 12 ounce cup of coffee at time  $t$  (in minutes since the coffee was brewed) is given by  $T(t) = \frac{3}{8}t^2 - 13t + 180$  for  $0 \leq t \leq 10$ . What is the most appropriate interpretation of the statement

$$T'(4.2) = -9.85?$$

- 4.2 minutes after the coffee was brewed, the temperature of the coffee was 9.85 degrees Fahrenheit less than the initial brew temperature.
- 4.2 minutes after the coffee was brewed, the temperature of the coffee was changing at a rate of  $-9.85$  degrees Fahrenheit per minute.
- On average, the temperature of the coffee decreased by 9.85 degrees Fahrenheit each minute over the first 4.2 minutes since it was brewed.
- The temperature of the coffee decreased by 9.85 degrees Fahrenheit during the fourth minute after the coffee was brewed.
- The temperature of the coffee was approximately 132.01 degrees Fahrenheit exactly 4.2 minutes after the coffee was brewed.

- (iv) Suppose  $f'(a) > \frac{f(a + \Delta x) - f(a)}{\Delta x}$  for  $\Delta x > 0$ . Which of the following could be a graph of  $f$ ?



- I only
  - II only
  - II and III only
  - I and IV only
  - III and IV only
- (v) If  $f$  is a differentiable function and  $a$  is a number, then  $f'(a)$  is given by which of the following expressions:

I.  $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

II.  $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

III.  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{x - h}$

- I only
- II only
- I and II only
- I and III only
- I, II, and III

2. (3 points each) Suppose a candle is burning at a constant rate of 1.3 inches per hour. Let  $g(t)$  represent the length of the candle (in inches) and let  $t$  represent the number of hours elapsed since the candle was lit.

(a) Explain what the expression  $g(1.3) - g(2.7)$  represents in the context of this situation.

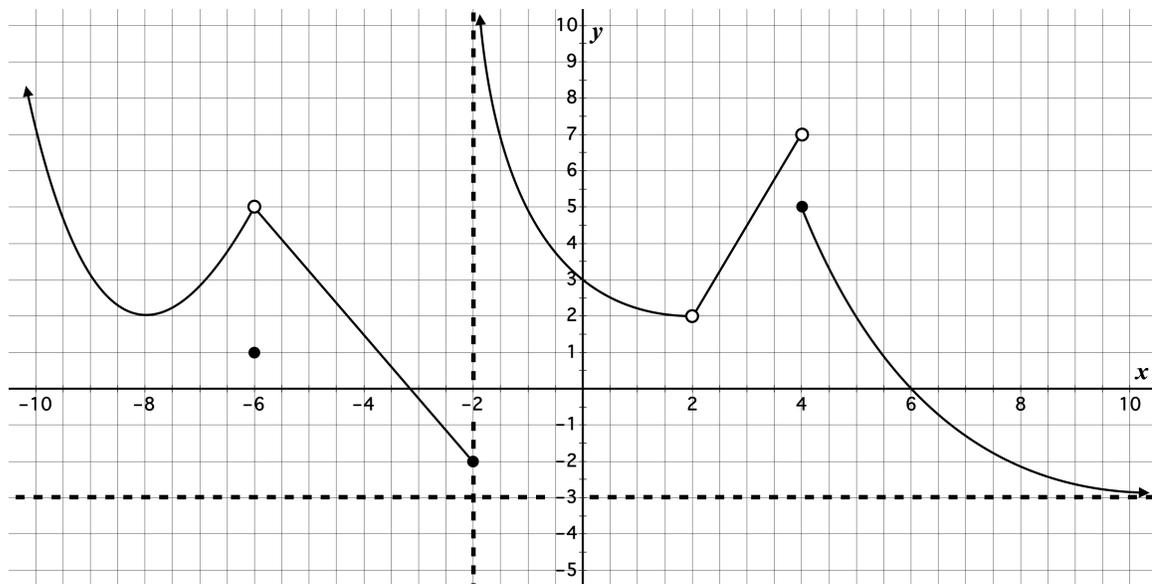
(b) Explain what the solution to the equation  $g(t) = 4.5$  represents in the context of this situation.

(c) Explain what the equation  $g^{-1}(2.398) = 3.44$  means in the context of this situation.

(d) Suppose that the candle is 3.1 inches long after having burned for 2.9 hours. What was the length of the candle before it was lit?

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3. (a) (9 points) Answer the following questions based on the graph of  $f$  below. Assume that all critical points, points of discontinuity, and points of inflection of  $f$  can be observed from the graph below. Asymptotes are indicated by dotted lines.



Give numeric values for each of the following. Write “DNE” if the value does not exist.

$$\lim_{x \rightarrow -6} f(x) =$$

$$f'(3) =$$

$$\lim_{x \rightarrow -2^-} f(x) =$$

$$\lim_{\Delta x \rightarrow 0} \frac{f(-8+\Delta x) - f(-8)}{\Delta x} =$$

$$\lim_{x \rightarrow 4} f(x) =$$

$$\left. \frac{df}{dx} \right|_{x=-4} =$$

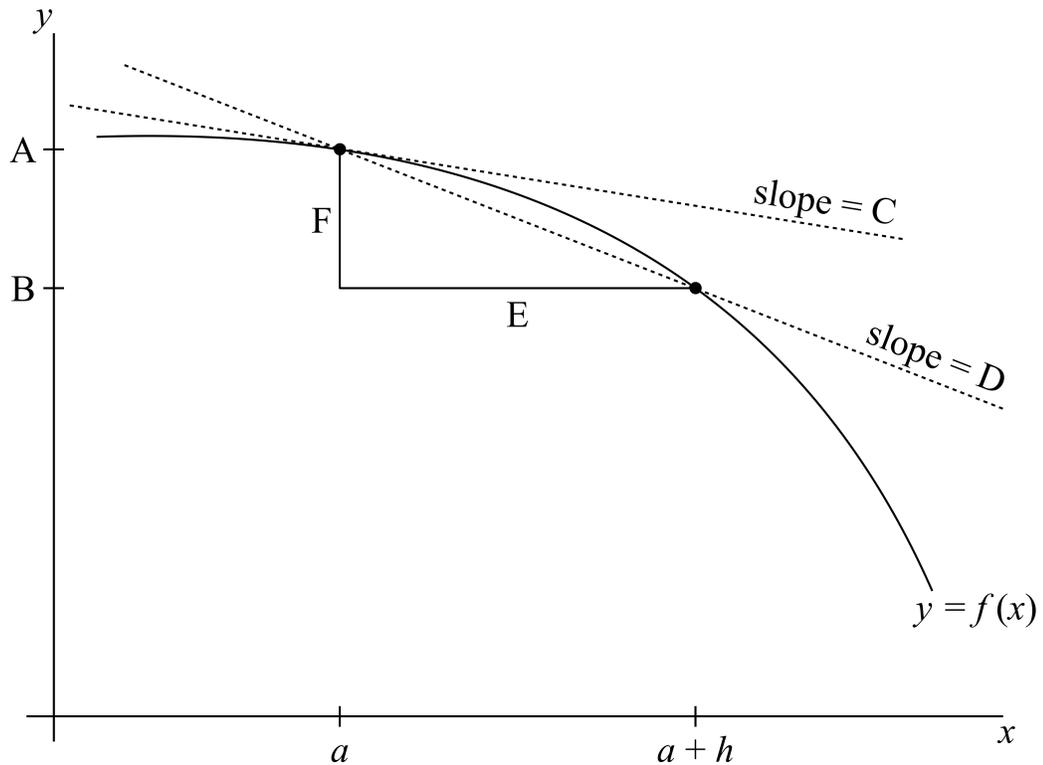
$$\lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} =$$

$$\lim_{x \rightarrow 2} f(x) =$$

$$\lim_{x \rightarrow 0} f(x) =$$

- (b) (2 points) Identify all  $x$ -values in the interval  $[-10, 10]$  where  $f$  is not differentiable.

4. (6 points) Consider the graph of  $y = f(x)$  illustrated below.



Write each graphical quantity, A-F, in the blank next to corresponding expression on the left. **Each letter will be used exactly once.**

<u>Expression</u>	<u>Graphical Quantity</u>
$h$	_____ A
$f(a)$	_____ B
$f(a+h)$	_____ C
$f(a+h) - f(a)$	_____ D
$\frac{f(a+h) - f(a)}{h}$	_____ E
$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$	_____ F

5. (3 points each) Let  $f$  be a continuous function that has the following known values.

$x$	-3	-2	-1	0	1	2	3
$f(x)$	-2	3.5	6	1	-0.5	-2	-4

(a) Does  $f(x) = 4.2$  have a solution? Justify your response.

(b) What is the minimum number of zeros that  $f(x)$  must have? Justify your response.

(c) Use the table of values to approximate  $f'(2)$ . Justify your response.

6. (a) (6 points) Use the limit definition of derivative to show that  $\frac{d}{dx}(3x^2 - 2) = 6x$ . You **must** use the **definition of derivative** to receive any credit.

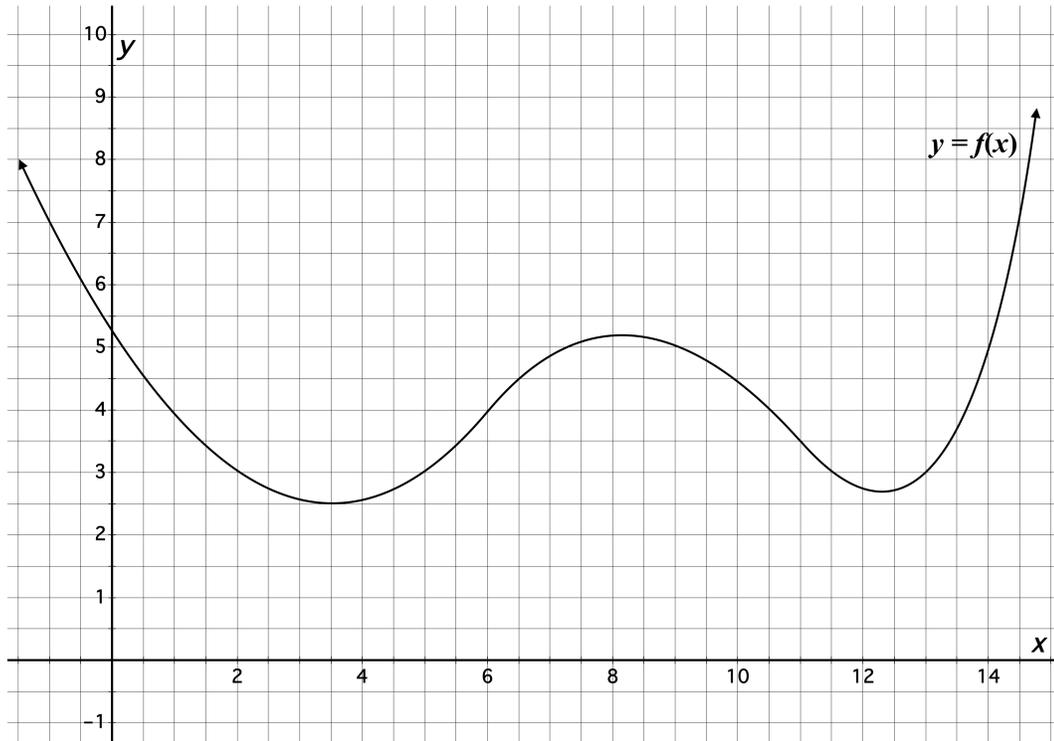
- (b) (3 points each) Compute the following derivatives. (You **may** use the Power Rule to compute these derivatives.)

i. Let  $f(x) = 4x^5 - 3x^2$ . Find  $f'(x)$ .

ii. Let  $y = \frac{1}{x^3} + \sqrt{x}$ . Find  $\frac{dy}{dx}$ .

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7. (2 points each) The graph of the function  $y = f(x)$  is given below. Determine values for constants  $a$  and/or  $b$  that make each of the following inequalities true.



(a)  $\frac{f(8) - f(a)}{8 - a} < 0$

$a =$

(b)  $0 < \frac{f(b+2) - f(b)}{2} < 1$

$b =$

(c)  $\frac{f(b) - f(a)}{b - a} < -1$

$a =$

$b =$