INSTRUCTIONS: This exam is a closed book exam. You may not use your text, homework, or other aids except for a 3 x 5-inch notecard. You may use an allowable calculator, TI-83 or TI-84 to
- perform operations on real numbers,
- evaluate functions at specific values, and
- look at graphs and/or tables.

A TI-89, TI-Nspire, or a calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are not allowed. Unless otherwise stated, you must show all of your work including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that correct answers using incorrect reasoning may not receive any credit. This comprehensive exam assesses your understanding of material covered throughout the entire course.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 12 problems on 15 pages. Make sure all problems and pages are present.

The exam is worth 150 points in total.

You have 1 hour and 50 minutes to work starting from the signal to begin. Good luck!
<table>
<thead>
<tr>
<th>No.</th>
<th>Out of</th>
<th>Pts.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>150</td>
<td></td>
</tr>
</tbody>
</table>
1. (2 points each) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

(i) The domain of the function $f$ is $x \geq 0$. The horizontal line $y = 2$ is an asymptote for the graph of the function $f$. Which of the following statements must be true?
   (a) $f(0) = 2$
   (b) $f(x) \neq 2$ for all $x \geq 0$.
   (c) $f(2)$ is undefined
   (d) $\lim_{x \to 2} f(x) = \infty$
   (e) $\lim_{x \to \infty} f(x) = 2$

(ii) If $\lim_{x \to a} f(x) = L$, which of the following must be true?
   I. $f(a) = L$
   II. $\lim_{x \to a^-} f(x) = L$
   III. $\lim_{x \to a^+} f(x) = L$
   (a) I only
   (b) I and II
   (c) I and III
   (d) II and III
   (e) I, II, and III

(iii) Suppose $f''(x) < 0$ for $2 \leq x \leq 7$. Let $L(x)$ represent the linearization of $f$ at $x = 2$. Which of the following is true?
   (a) $L(2.1)$ is an underestimate of the value of $f(2.1)$.
   (b) $L(2.1)$ is an overestimate of the value of $f(2.1)$.
   (c) $L(2.1)$ is equal to the value of $f(2.1)$.
   (d) There is not enough information to compare the value of $L(2.1)$ to the value of $f(2.1)$.

(iv) If $f$ is a continuous function and if $F'(x) = f(x)$ for all real numbers $x$, then
   \[ \int_{-1}^{2} f(3x)dx = \]
   (a) $3F(2) - 3F(-1)$
   (b) $\frac{1}{3}F(2) - \frac{1}{3}F(-1)$
   (c) $F(6) - F(-3)$
   (d) $3F(6) - 3F(-3)$
   (e) $\frac{1}{3}F(6) - \frac{1}{3}F(-3)$
(v) The function \( y = f(x) \) measures the U.S. production of natural gas (in millions of cubic feet) at time \( x \) where \( x \) is measured in years since January 1, 1970. What is the most appropriate interpretation of \( f'(35) = 579 \)?

(a) On January 1, 2005, U.S. natural gas production was growing at a rate of 579 million cubic feet per year.


(c) U.S. natural gas production was 579 million cubic feet in 2005.

(d) On average, U.S. natural gas production increased by 579 million cubic feet per year over the first 35 years following 1970.

(e) U.S. natural gas production grew by 579 million cubic feet from 2004 to 2005.

(vi) Ana met with some friends at District Bicycles in downtown Stillwater to go on a bike ride. Let \( v(t) \) represent Ana’s velocity (in miles per hour) \( t \) hours after she left the bike shop. Which of the following best describes the meaning of \( \int_{0.5}^{2} v(t) dt \)?

(a) The change in Ana’s velocity from 0.5 hours to 2 hours after she left the bike shop.

(b) Ana’s average speed from 0.5 hours to 2 hours after she left the bike shop.

(c) The change in Ana’s distance away from the bike shop from 0.5 hours to 2 hours after she left the bike shop.

(d) Ana’s distance away from the bike shop 1.5 hours after she left the bike shop.

(e) The sum of Ana’s velocity 0.5 hours after she left the bike shop and her velocity 2 hours after she left the bike shop.
2. (9 points) Answer the following questions based on the graph of $f$ below. Assume that all critical points, point of discontinuity, and points of inflection of $f$ can be observed from the graph below. Asymptotes are indicated by dotted lines.

Give numeric values for each of the following. Write “DNE” if the value does not exist and “$\infty$” or “$-\infty$” as appropriate.

\[
\begin{align*}
f(-2) &= \\
f(5) &= \\
\lim_{x \to -2} f(x) &= \\
\lim_{x \to -\infty} f(x) &= \\
\lim_{x \to 3^-} f(x) &= \\
\lim_{x \to 5^+} f(x) &= \\
f''(2) &= \\
f'(1) &= 
\end{align*}
\]
3. (5 points each) Evaluate the following limits or state that they do not exist (“DNE”). Use \( \infty \) or \( -\infty \) if either is appropriate. Numbers alone without justification (either algebra and/or quoting theorems where applicable) will receive no credit. **If you use L’Hôpital’s Rule, clearly indicate when you do so.**

(a) \( \lim_{x \to 8} \frac{\sqrt{x} - 4 - 2}{x - 8} = \)

(b) \( \lim_{x \to \infty} \frac{e^{2x} - 1}{x^2} = \)

(c) \( \lim_{t \to -2} \frac{t^2 + 3t + 2}{t + 2} = \)

(d) \( \lim_{\theta \to \pi} \tan(\theta) \csc(\theta) = \)
4. (5 points each) Use calculus to determine the following for the function
\( f(x) = x^3 + x^2 - x. \)

(a) Find the critical points of \( f \). State your answers as exact values. (Work must be shown to receive credit.)

(b) Determine the absolute maximum and absolute minimum of \( f \) on the interval \([-2, 2]\). Round your answers to two decimal places. (Work must be shown to receive credit.)

(c) Determine the coordinates of all inflection points of \( f \) on the interval \([-2, 2]\). (Work must be shown to receive credit.)
5. (3 points each) Let \( f(x) \) be a continuous function with the following known values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-2</th>
<th>-1</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>-5</td>
<td>-1</td>
<td>1</td>
<td>-3</td>
<td>4</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

(a) What is the minimum number of zeros that \( f(x) \) must have? Justify your response.

(b) Does \( f(x) = -3.5 \) have a solution? Justify your response.
6. (5 points each) Compute the following derivatives. Do not simplify.

(a) Let \( y = \frac{2^x}{x^3 - 1} \). Find \( \frac{dy}{dx} \).

(b) Let \( f(t) = t \sin(3t) \). Find \( \frac{df}{dt} \).

(c) Let \( g(x) = \ln(x^2) \). Find \( g''(x) \).

(d) Find \( \frac{dy}{dx} \) for the curve \( \sin(y) = x + \cos(y) \). (You must solve for \( \frac{dy}{dx} \) in terms of \( x \) and \( y \)).
7. (10 points) An airplane is flying at an altitude of 8 kilometers and passes directly over a radar antenna. When the distance between the plane and the antenna is 12 kilometers, the radar detects that the distance between the plane and the antenna is changing at a rate of 340 kilometers per hour. What is the speed of the airplane at that moment? (Round your answer to three decimal places.)
8. (10 points) We want to construct a rectangular box with a square base as shown below. The box will have a surface area of 12 square feet. Use calculus to determine the maximum volume of the box. Write your solution as an exact value, not a decimal approximation.
9. (5 points each) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

(a) \( \int_{0}^{\pi} \cos(\theta) \theta^3 \sin(\theta) d\theta = \)

(b) \( \int (x + \sec^2(x)) \, dx = \)

(c) \( \frac{d}{dx} \int_{1}^{x} (t^5 - 7t^3) \, dt = \)

(d) \( \int \frac{\ln(x)}{x} \, dx = \)
10. (8 points) The function $A$ is defined as $A(x) = \int_{-4}^{x} f(t)dt$. The graph of $f$ is shown below.

(a) (2 points) Shade the region corresponding to $A(1)$ on the graph above.

(b) (2 points) Evaluate $A(1)$.

(c) (2 points) Evaluate $f'(2)$.

(d) (2 points) Give the $x$–coordinates of all critical points of $A$ on the interval $(-5, 5)$. 
11(a) (6 points) Determine the area of the region between the graphs of $y = 2x + 4$ and $y = x^2 + 4$ over the interval $[0, 2]$.

11(b) (6 points) Suppose the region between the graphs of $y = 2x + 4$ and $y = x^2 + 4$ over the interval $[0, 2]$ is rotated around the line $y = 3$. Completely set up the integral that represents the volume of the resulting solid but do not evaluate this integral.
12 (8 points) The graphs below are of the function $g$. Write an expression in each blank that represents the numerical value of the quantity identified on the corresponding graph. Select the appropriate expression for each blank from only the options below.

\[
\frac{d}{dx} \int_2^x g(t)dt \quad \frac{g(4) - g(0)}{4 - 0} \quad \lim_{x \to 2} g(x) \quad \int_1^4 g(x)dx \quad R_6 \quad \lim_{h \to 0} \frac{g(2 + h) - g(2)}{h}
\]

\[
\int g(x)dx \quad \lim_{h \to 0} \frac{g(x + h) - g(x)}{h} \quad L_6 \quad \frac{g(4) - g(1)}{4 - 1} \quad \frac{dg}{dx} \quad \sum_{i=1}^6 g(x)
\]

(a) Slope of the line tangent to the graph of $g$ at $(2, 2)$.

Expression: ____________________

(b) Area bound by the graph of $g$ and the $x$-axis on the interval $[1, 4]$.

Expression: ____________________

(c) Sum of the area of the rectangles shown below.

Expression: ____________________

(d) Slope of the line secant to the graph of $g$ which passes through the points $(0, 1)$ and $(4, 4)$.

Expression: ____________________