FINAL EXAM INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a 3×5 notecard. You may use an allowable calculator, **TI 83 or 84** to

- perform operations on real numbers,
- evaluate functions at specific values, and
- look at graphs.

A TI 89, Nspire, or a calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that **correct answers using incorrect reasoning may not receive any credit**. Reasoning which will earn credit will use material covered in the course. Some short-cuts, such as the using the Fundamental Theorem of Calculus to evaluate a definite integral when the problem asks you to use Riemann sums, may not receive any credit.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 15 problems on the following 16 pages. Make sure all problems and pages are present.

The final exam is worth 250 points in total.

You have **1 hour** and **50 minutes** to work starting from the signal to begin. Good luck!

No.	**Out of**	**Pts.**	**No.**	**Out of**	**Pts.**
1	12		8	14	
2	33		9	9	
3	16		10	14	
4	9		11	14	
5	18		12	30	
6	11		13	18	
7	26		14	12	
			15	14	
			Exam	250	

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1. (*3 points each*) True (T) or False (F) statements. Circle your answer. No justification needed. True means always true.

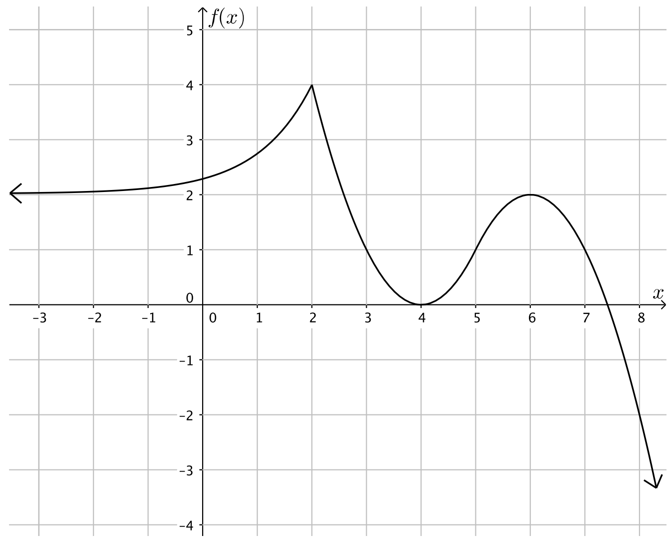
(a) **T** **F** If $\lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x)$, then f is continuous at $x = 5$.

(b) **T** **F** If $f''(x) > 0$ on (-2,5), then on this interval f must be increasing.

(c) **T** **F** If f and g are differentiable functions where $f(x) > g(x)$ for all x in the interval $[0, 1]$, then $f'(x) > g'(x)$ for all x in the interval $[0, 1]$.

(e) **T** **F** If f is a continuous function on $[1, 4]$ such that $f(x) \geq 2$ on $[1, 4]$, then $\int_1^4 f(x)dx \geq 6$.

2. (*33 points*) Answer the following questions based on the graph of f below. Assume that the domain of f is $(-\infty, \infty)$ and that the end behavior of f continues as indicated by the graph.



(a) Give numeric values for each of the following. Write "DNE" if the value does not exist and "∞" or "$-\infty$" if appropriate.

$f(3) =$ $\qquad$ $\lim_{x\to 3} f(x) =$ $\qquad$ $\lim_{x\to -\infty} f(x) =$

$\lim_{x\to \infty} f(x) =$ $\qquad$ $\lim_{x\to 2^+} f(x) =$ $\qquad$ $\lim_{x\to 2} f(x) =$

(b) List all x-values (if any) where there is a critical point. Write "DNE" if no such x-values exist.

(c) List all x-values (if any) where there is an inflection point. Write "DNE" if no such x-values exist.

3. (*8 points each*) Evaluate the following limits as exact numbers, or state that it does not exist. Use "∞" or "$-\infty$" if either is appropriate.
DO NOT use numerical estimation or graphing to find the limits.
IF you use L'Hôpital's Rule, clearly indicate when doing so.

(a) $\displaystyle\lim_{\theta\to 0}\frac{\sin(3\theta)}{\sin(7\theta)}$

(b) $\displaystyle\lim_{x\to 2}\frac{\sqrt{x^2+12}-4}{x-2}$

4. (*9 points*) Using the appropriate limits, find ALL horizontal asymptotes of

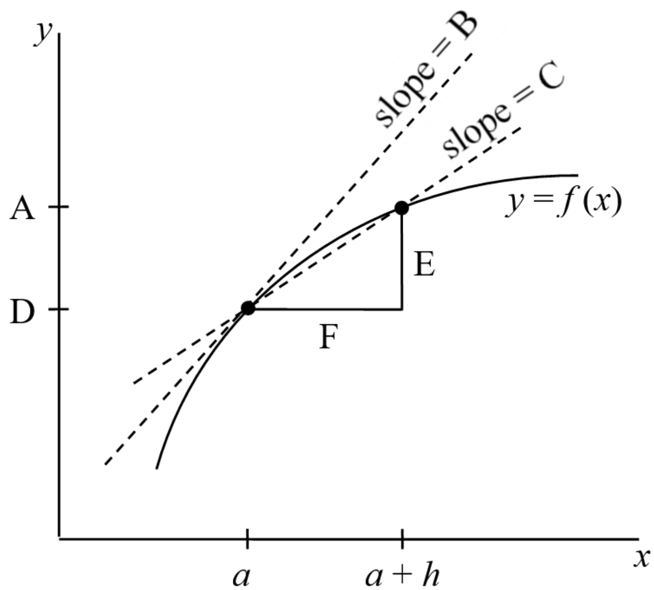
$$f(x) = \frac{1 - e^x}{1 + 2e^x}$$

You must show all your work using limits to earn any credit. IF you use L'Hôpital's Rule, clearly indicate when doing so.

5. (*3 points each*) Match the graphical quantities A-E with the appropriate expressions from the definition of the derivative

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Use the quantities marked on the graph of $y = f(x)$ as illustrated below.



Match each graphical quantity in the left column with with appropriate expression in the right column by drawing a line connecting a letter on the left with exactly one expression on the right. Each expression will be used exactly once.

A	h
B	$f(a)$
C	$f(a+h)$
D	$f(a+h) - f(a)$
E	$\dfrac{f(a+h) - f(a)}{h}$
F	$\lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

6. (11 points) A smoke jumper jumps out of an airplane to fight a large forest fire. The distance fallen by this smoke jumper t seconds after she opens her parachute is given by $d(t)$ feet. Her distance fallen at various times is shown in the following table.

t seconds	0	0.5	1	1.5	2	2.5	3	3.5	4
$d(t)$ feet	0	31	61	90	118	144	166	184	195

(a) Using only data from the table, find the best possible approximation to $d'(2)$ using an interval of time with only times ≤ 2 seconds.

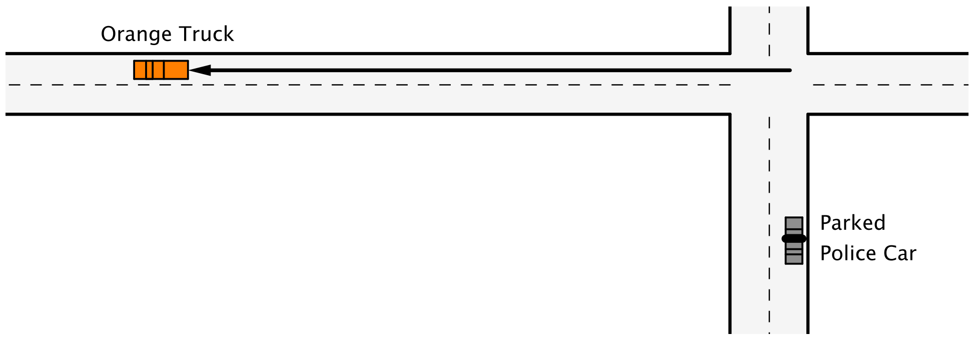
(b) From the moment her parachute opened (0 seconds) to the 4^{th} second, the speed of her fall was decreasing. Based on this information, is your approximation in part (a) an overestimate or an underestimate to $d'(2)$?

7. (*13 points each*) For each of the following, find the derivative. Do not simplify.

   (a) $f(x) = \dfrac{3}{x^5} + \tan(7x) - 8$

   (b) $P(t) = 7\big(\sin t\big)\big(e^{2t}\big)$

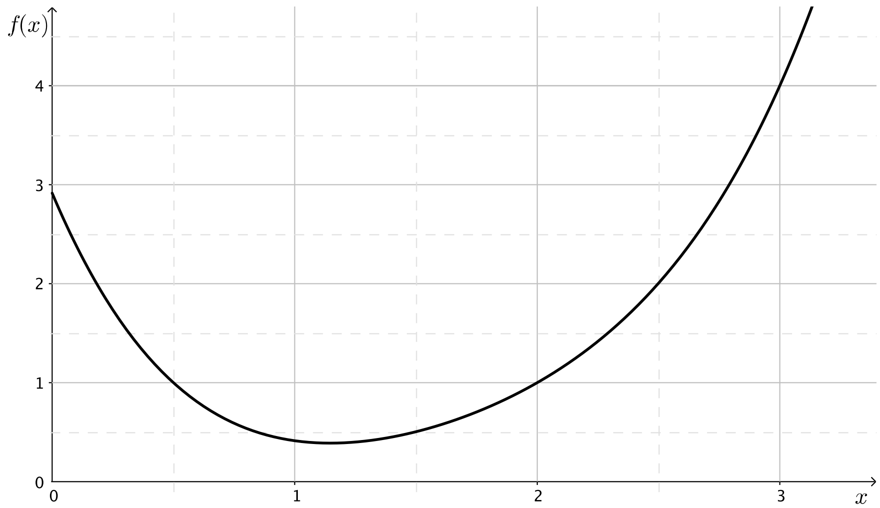
8. (*14 points*) Pistol Pete has been seen leaving an intersection heading due west in an orange pickup truck. A police car is parked 0.1 miles due south of the intersection. When the distance between the stationary police car and the moving orange truck is 0.6 miles, the distance between the police car and the truck is increasing at 55 mph. How fast is Pistol Pete driving at this time?



9. (*9 points*) Write a formula for the linearization, $L(x)$, of $f(x) = \sqrt{x}$ at $a = 9$.

10. (*14 points*) A farmer has 1200 ft of fencing and wants to fence off a rectangular field that borders a barn. He does not need any fence next to the barn. What are the dimensions of the field that has the largest area? You must show your work using calculus to answer the question AND use calculus to justify that your answer gives a maximum area.

11. (*14 points*) Let f be the function shown in the graph below.



(a) Use R_4 to approximate the area under the graph of f over the interval $[1, 3]$.

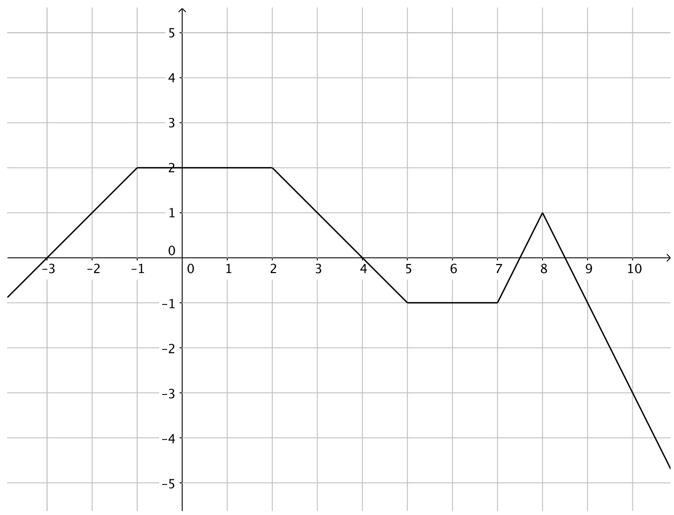
(b) Draw the rectangles representing R_4 on the graph above. Is your estimation an overestimate or an underestimate of $\int_1^3 f(x)\,dx$?

12. (*15 points each*) Evaluate the following integrals. Show ALL of your work.

   (a) $\displaystyle\int_1^4 \left(\frac{1}{x^2} + 3\sqrt{x}\right) dx$

   (b) $\displaystyle\int \frac{(\ln(x))^2}{5x}\, dx$

13. (*6 points each*) Let f be the function whose graph is given below.



Define a function $A(x)$ by

$$A(x) = \int_1^x f(t)\, dt.$$

(a) On the graph above, shade in the region corresponding to $A(5)$.

Evaluate the following:

(b) $A(5) =$

(c) $A'(6) =$

14. (*12 points*) Completely set up the integral that represents the area of the region enclosed by $y = \cos x$ and $y = 2 - \cos x$ over the interval $[0, 2\pi]$. DO NOT EVALUATE this integral. If you have any questions about what "completely set up" might mean, do not hesitate to ask your instructor.

15. (*14 points*) Suppose the region between $y = x$ and $y = x^3$ on $[0, 1]$ is rotated around $y = -1$. Completely set up the integral that represents the volume of the resulting solid but DO NOT EVALUATE this integral.