INSTRUCTIONS: This exam is a closed book exam. You may not use your text, homework, or other aids except for a 3 × 5-inch notecard. You may use an allowable calculator, TI-83 or TI-84 to:

- perform operations on real numbers,
- evaluate functions at specific values, and
- look at graphs and/or tables.

A TI-89, TI-Nspire, or a calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are not allowed. Unless otherwise stated, you must show all of your work including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that correct answers using incorrect reasoning may not receive any credit. This comprehensive exam assesses your understanding of material covered throughout the entire course.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 10 problems on 14 pages. Make sure all problems and pages are present.

The exam is worth ? points in total.

You have 1 hour and 50 minutes to work starting from the signal to begin. Good luck!
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1. (3 points each) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (i) Consider the function \( f \) defined by
   \[
   f(x) = \begin{cases} 
   3x^2 - 4, & x < 1 \\
   2, & x = 1 \\
   6x - 7, & x > 1
   \end{cases}
   \]

   Which of the following are true statements about this function?
   I. \( \lim_{x \to 1} f(x) \) exists
   II. \( f(1) \) exists
   III. \( f \) is continuous at \( x = 1 \)
   a. I only
   b. I and II
   c. II only
   d. II and III
   e. I, II, and III

   (ii) The expression \( \lim_{h \to 0} \frac{(x + h)^3 - \ln(x + h) - (x^3 - \ln(x))}{h} \) is the derivative of what function?
   a. \( f(x) = (x + h)^3 - \ln(x + h) \)
   b. \( f(x) = 3x^2 - \frac{1}{x} \)
   c. \( f(x) = (x + h)^3 - \ln(x + h) - (x^3 - \ln(x)) \)
   d. \( f(x) = x^3 - \ln(x) \)
   e. \( f(x) = \frac{(x + h)^3 - \ln(x + h) - (x^3 - \ln(x))}{h} \)

   (iii) If \( f \) and \( g \) are continuous functions with \( f(5) = 5 \) and \( \lim_{x \to 5} (2f(x) - g(x)) = 6 \), find \( g(5) \).
   a. \( g(5) = 4 \)
   b. \( g(5) = 5 \)
   c. \( g(5) = 16 \)
   d. \( g(5) = 2 \)
   e. \( g(5) = 6 \)
(iv) If \( g'(t) \) represents a child’s rate of growth in pounds per year, which of the following expressions represents the increase in the child’s weight (in pounds) between years 2 and 5?

a. \( \int_{2}^{5} g'(t) \, dt \)

b. \( g'(5) - g'(2) \)

c. \( \int_{5}^{2} g'(t) \, dt \)

d. \( \frac{g'(5) - g'(2)}{5 - 2} \)

e. None of these expressions represents the increase in the child’s weight (in pounds) between years 2 and 5.

(v) A portion of the graph of a function \( f \) that passes through the point \( (c, f(c)) \) is shown below. Which of the following conditions are satisfied?

\[
\begin{align*}
&f'(c) > 0, \quad f''(x) > 0 \text{ for all } x < c, \quad \text{and } f''(x) < 0 \text{ for all } x > c \\
&f'(c) < 0, \quad f''(x) < 0 \text{ for all } x < c, \quad \text{and } f''(x) > 0 \text{ for all } x > c
\end{align*}
\]

e. None of the above.
2. (9 points) Answer the following questions based on the graph of \( f \) below. Assume that all critical points, points of discontinuity, and points of inflection of \( f \) can be observed from the graph below. Asymptotes are indicated by dotted lines.

Give numeric values for each of the following. Write “DNE” if the value does not exist and “\( \infty \)” or “\(-\infty\)” as appropriate.

\[
\lim_{x \to -6} f(x) = \quad f'(-8) = \quad \lim_{x \to -2^+} f(x) =
\]

\[
\int_{2}^{4} f(x) \, dx = \quad \lim_{x \to 4} f(x) = \quad \lim_{\Delta x \to 0} \frac{f(3 + \Delta x) - f(3)}{\Delta x} =
\]

\[
\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \quad \lim_{x \to \infty} f(x) = \quad \lim_{x \to 0} f(x) =
\]
3. (5 points each) Evaluate the following limits or state that they do not exist (“DNE”). Use $\infty$ or $-\infty$ if either is appropriate. Numbers alone without justification (either algebra and/or quoting theorems where applicable) will receive no credit. If you use L'Hôpital’s Rule, clearly indicate when you do so.

(a) \[ \lim_{x \to -3} \frac{x + 3}{x^2 + x - 6} = \]

(b) \[ \lim_{x \to 1} \frac{x^4 - 1}{x^3 - 1} = \]

(c) \[ \lim_{\theta \to \frac{\pi}{2}} \frac{\cot(\theta)}{\csc(\theta)} = \]
4. (14 points) The graph of the function $f$ is shown below. Use this graph for answering parts (b) and (c).

(a) (4 points) Suppose $f$ is a function and suppose $H(x) = \frac{f(x)}{x}$. Compute $H'(x)$ in terms of $f$ and $f'$. (Note: You do not need to refer to the graph above to answer this question.)

(b) (4 points) Refer to the graph of $f$ above to evaluate $H'(1)$.

(c) (2 points each) Define $A(x)$ by $A(x) = \int_{0}^{x} f(t) \, dt$. Refer to the graph of $f$ above to evaluate the following:

i. $A(-2) =$

ii. $A'(2) =$

iii. $A''(-1) =$
5. (5 points each) Compute the following derivatives. Do not simplify.

(a) Let \( y = \cos^2(3x) \). Find \( \frac{dy}{dx} \).

(b) Let \( f(t) = 4e^{-t} + 5e^t \). Find \( f^{(4)}(t) \).

(c) Let \( g(x) = \ln(x) \sec(x) \). Find \( g'(x) \).

(d) Let \( f(x) = e^x + x^e + ex \). Find \( \frac{df}{dx} \).
6. (6 points) Find \( \frac{dy}{dx} \) for the curve \( x^2 + \sin(y) = y^2 + 1 \). (You must solve for \( \frac{dy}{dx} \) in terms of \( x \) and \( y \)).
7. (8 points) A 6 foot-tall man is walking away from a lamp post, which is 11 feet tall. Let $x$ represent the man's distance from the lamp post (in feet) and let $y$ represent the length of the man's shadow (in feet), as shown in the picture below. Determine how fast the man is walking if the length of his shadow is increasing at a rate of 12 feet per second.
8. (8 points) **Use calculus** to find the dimensions of the rectangle of largest area that can be inscribed in the region bounded by $x + y = 1$, and the positive $x$ and $y$ axes.
9. (5 points each) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

(a) \[ \int \frac{x}{\sqrt{x^2 + 5}} \, dx = \]

(b) \[ \int_{-2}^{3} e^{5x} \, dx = \]

(c) \[ \frac{d}{dx} \int_{1}^{x^2} (t + 2) \, dt = \]
10(a) (6 points) Determine the area of the region bound by the graphs of $f(x) = 2x$ and $g(x) = x^{3/2}$ over the interval $[0, 4]$.

10(b) (4 points) Suppose the region between the graphs of $f(x) = 2x$ and $g(x) = x^{3/2}$ over the interval $[0, 4]$ is rotated around the $x$-axis. Completely set up the integral that represents the volume of the resulting solid but **do not evaluate this integral**.
BASIC FORMULAS

\[ \frac{d}{dx} x^n = nx^{n-1} \]
\[ \frac{d}{dx} e^x = e^x \]
\[ \frac{d}{dx} \ln(x) = \frac{1}{x} \]
\[ \frac{d}{dx} \log_a(x) = \frac{1}{x \ln(a)} \]
\[ \frac{d}{dx} a^x = a^x \ln(a) \]
\[ \frac{d}{dx} \sin(x) = \cos(x) \]
\[ \frac{d}{dx} \cos(x) = -\sin(x) \]
\[ \frac{d}{dx} \tan(x) = \sec^2(x) \]
\[ \frac{d}{dx} \cot(x) = -\csc^2(x) \]
\[ \frac{d}{dx} \sec(x) = \sec(x) \tan(x) \]
\[ \frac{d}{dx} \csc(x) = -\csc(x) \cot(x) \]
\[ \frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \]
\[ \frac{d}{dx} \cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}} \]
\[ \frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2} \]
\[ \frac{d}{dx} \cot^{-1}(x) = -\frac{1}{1+x^2} \]
\[ \int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1 \]
\[ \int \frac{du}{u} = \ln|u| + C \]
\[ \int e^u du = e^u + C \]
\[ \int a^u du = \frac{a^u}{\ln a} + C \]
\[ \int \sin(u) du = -\cos(u) + C \]
\[ \int \cos(u) du = \sin(u) + C \]
\[ \int \sec^2(u) du = \tan(u) + C \]
\[ \int \csc^2(u) du = -\cot(u) + C \]
\[ \int \sec(u) \tan(u) du = \sec(u) + C \]
\[ \int \csc(u) \cot(u) du = -\csc(u) + C \]
\[ \int \tan(u) du = \ln|\sec u| + C \]
\[ \int \cot(u) du = \ln|\sin u| + C \]
\[ \int \sec(u) du = \ln|\sec u + \tan u| + C \]
\[ \int \csc(u) du = \ln|\csc u + \cot u| + C \]
\[ \int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \]
\[ \int \frac{du}{a^2+u^2} = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C \]
\[ \int f(u(x)) \cdot u'(x) dx = \int f(u) du \]
\[ L_n = \Delta x \sum_{k=1}^{n} f \left(a + (k - 1)\Delta x\right) \]
\[ R_n = \Delta x \sum_{k=1}^{n} f \left(a + k\Delta x\right) \]