INSTRUCTIONS: This exam is a **closed book exam**. You may **not** use your text, homework, or other aids except for a 3×5-inch notecard. You may use an allowable calculator, **TI-83 or TI-84** to

- perform operations on real numbers,
- evaluate functions at specific values, and
- look at graphs and/or tables.

A TI-89, TI-Nspire, or a calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are **not allowed**. Unless otherwise stated, you must **show all of your work** including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that **correct answers using incorrect reasoning may not receive any credit**. This exam assesses your understanding of material covered up to and including Section 5.6 of Rogawski and Adams.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 7 problems on 9 pages. Make sure all problems and pages are present.

The exam is worth 65 points in total.

You have **60 minutes** to work starting from the signal to begin. Good luck!

Exam 3 Grade by Problem Number

No.	Out of	Pts.
1	10	
2	6	
3	6	
4	12	
5	9	
6	12	
7	10	
Total	65	

Current Course Grade by Category

Category	Out of	Current
Exam 1	100%	
Exam 2	100%	
Exam 3	100%	
WebAssign	100%	
Labs/Quiz/HW	100%	
Overall 14 Week Grade	100%	

1. (*2 points each*) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

   (i) Let $r(t)$ represent the rate at which water drains from a tank (in gallons per minute) and let t represent the number of minutes elapsed since water started draining from the tank. Which of the following best describes the meaning of $\int_1^4 r(t)\,dt$?

   (a) The average rate at which water drains from the tank from 1 minute to 4 minutes after water started draining from the tank.

   (b) The number of gallons of water drained from the tank 3 minutes after water started draining from the tank.

   (c) The change in the rate at which water drains from the tank from 1 minute to 4 minutes after water started draining from the tank.

   (d) The change in the number of gallons of water drained from the tank from 1 minute to 4 minutes after water started draining from the tank.

   (e) The rate at which water drains from the tank 3 minutes after water started draining from the tank.

   (ii) Let f be a positive, strictly increasing function on $[2,4]$. Which of the following approximations of $\int_2^4 f(x)\,dx$ is the largest?

   (a) R_4
   (b) L_4
   (c) M_4
   (d) They are all equal.
   (e) There is not enough information provided to determine which approximation is largest.

   (iii) Suppose f is an even function, $\int_{-2}^{2} f(x)\,dx = 6$, and $\int_{-5}^{5} f(x)\,dx = 14$. Find $\int_2^5 f(x)\,dx$.

   (*Note that a function f is even if $f(-x) = f(x)$ for all x in the domain of f.*)

   (a) 10
   (b) 8
   (c) 4
   (d) 20
   (e) There is not enough information provided to determine the value of $\int_2^5 f(x)\,dx$.

(iv) The expression $\displaystyle\sum_{k=1}^{10}\left(1+\frac{3k}{10}\right)^3\cdot\frac{3}{10}$ is a Riemann sum approximation for which of the following?

(a) $\displaystyle\frac{3}{10}\int_1^{10} x^3\,dx$

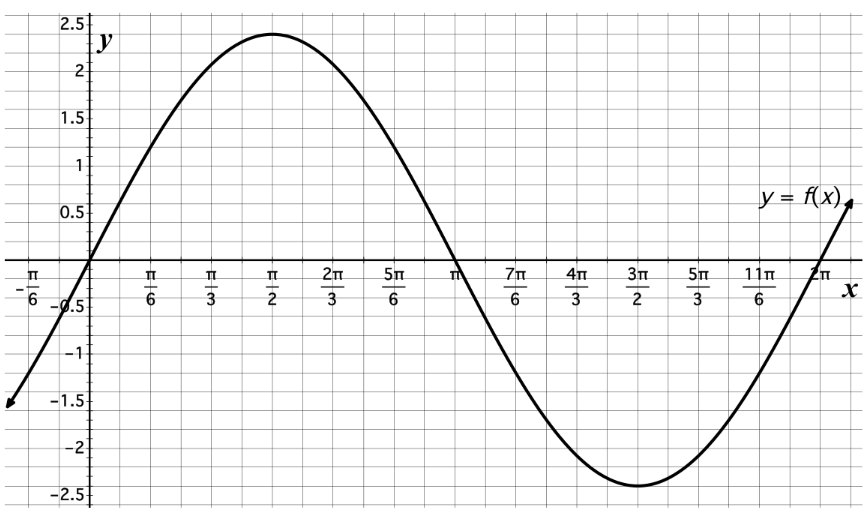
(b) $\displaystyle\int_1^{10}(1+x)^3\,dx$

(c) $\displaystyle\frac{3}{10}\int_1^{10}\left(1+\frac{3x}{10}\right)^3\,dx$

(d) $\displaystyle\int_1^{4} x^3\,dx$

(e) $\displaystyle\frac{3}{10}\int_1^{4} x^3\,dx$

(v) The graph of the function f is below. The following inequalities compare the values of the quantities $f'\left(\frac{\pi}{6}\right)$, $f''\left(\frac{\pi}{2}\right)$, $\dfrac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{6}\right)}{\frac{\pi}{2}-\frac{\pi}{6}}$, and $\displaystyle\int_0^{2\pi} f(x)\,dx$. Which of the following string of inequalities is true?



(a) $\displaystyle\int_0^{2\pi} f(x)\,dx > f''\left(\frac{\pi}{2}\right) > \frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{6}\right)}{\frac{\pi}{2}-\frac{\pi}{6}} > f'\left(\frac{\pi}{6}\right)$

(b) $\displaystyle f'\left(\frac{\pi}{6}\right) > \frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{6}\right)}{\frac{\pi}{2}-\frac{\pi}{6}} > \int_0^{2\pi} f(x)\,dx > f''\left(\frac{\pi}{2}\right)$

(c) $\displaystyle\frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{6}\right)}{\frac{\pi}{2}-\frac{\pi}{6}} > f'\left(\frac{\pi}{6}\right) > \int_0^{2\pi} f(x)\,dx > f''\left(\frac{\pi}{2}\right)$

(d) $\displaystyle f''\left(\frac{\pi}{2}\right) > \int_0^{2\pi} f(x)\,dx > \frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{6}\right)}{\frac{\pi}{2}-\frac{\pi}{6}} > f'\left(\frac{\pi}{6}\right)$

(e) $\displaystyle f'\left(\frac{\pi}{6}\right) > f''\left(\frac{\pi}{2}\right) > \int_0^{2\pi} f(x)\,dx > \frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{6}\right)}{\frac{\pi}{2}-\frac{\pi}{6}}$

2. (*6 points*) A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a fence. With 800 meters of fencing at your disposal, what is the largest area you can enclose? **Justify your response.**

3. (*6 points*) A man 6 feet tall walks at a constant rate of 5 ft/sec toward a streetlight that is 16 feet above the ground. Determine the constant rate at which the length of his shadow changing when as he walks towards the light.

4. (*4 points each*) Evaluate the following limits or state that they do not exist ("DNE"). Use ∞ or $-\infty$ if either is appropriate. Numbers alone without justification (either algebra and/or quoting theorems where applicable) will receive no credit. **If you use L'Hôpital's Rule, clearly indicate when you do so**.

   (a) $\displaystyle\lim_{t\to 0} \frac{e^t - 1 - t}{t^2} =$

   (b) $\displaystyle\lim_{\theta\to \pi} \frac{\sin^2(\theta)}{\theta - \pi} =$

   (c) $\displaystyle\lim_{x\to \infty} \frac{\ln(x)}{\sqrt{x}} =$

5. (*3 points each*) Coal gas is produced at a gasworks. Pollutants in the gas are removed by scrubbers, which become less and less efficient as time goes on. The following measurements, made twice each month, show the rate at which pollutants are escaping (in tons/month) in the gas. Note that R is an increasing function.

Time t in months	0	0.5	1	1.5	2	2.5	3
Rate $R(t)$ in tons/month	2	6	10	17	27	38	50

(a) Use the left endpoint approximation with three subintervals, L_3, to approximate the total amount of pollutants (in tons) that escape during the three-month time period.

(b) Is your approximation in part (a) an overestimate or an underestimate of the actual amount of pollutants (in tons) that escape during the three-month time period? **Explain your response**.

(c) Write an integral that represents the exact amount of pollutants (in tons) that escape during the three-month time period. **Do not evaluate this integral**.

6. (*4 points each*) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

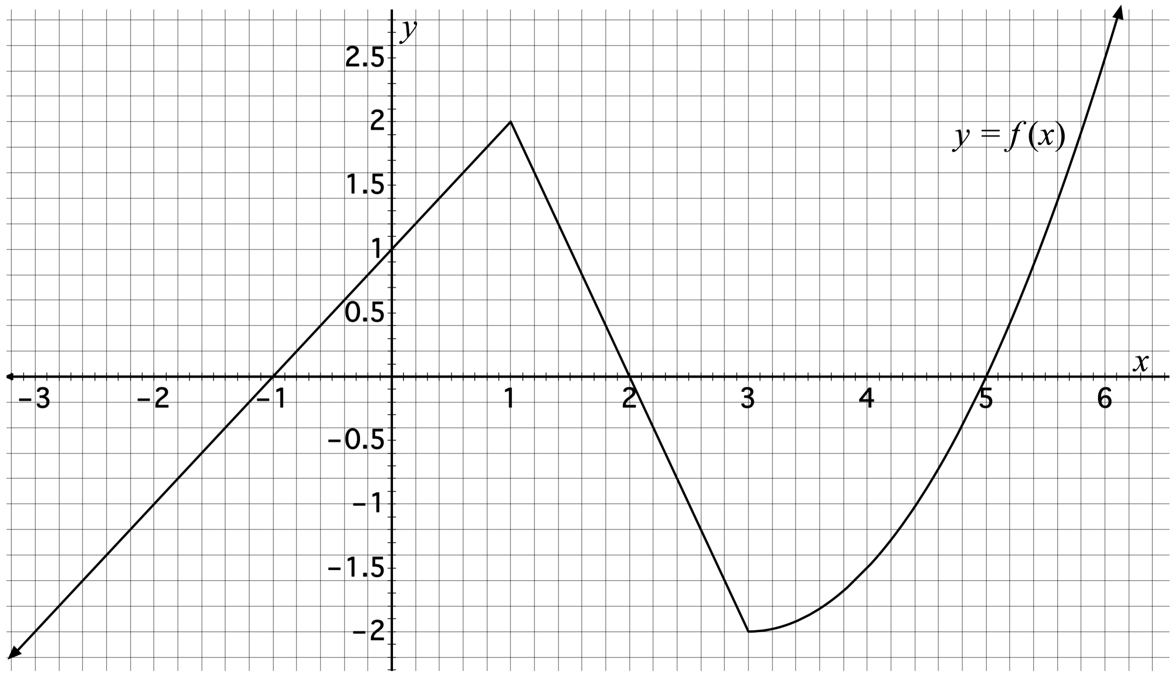
   (a) $\int \left(\frac{1}{\sqrt{x}} + \pi \sin(x) \right) \, dx =$

   (b) $\int_3^7 \left(e^x + 5 \right) \, dx =$

   (c) $\frac{d}{dx} \int_{\pi}^{x^3} \sec\left(t^2\right) \, dt =$

7. (*2 points each*) Let $y = f(t)$ be the function whose graph is given below, and define $F(x)$ by

$$F(x) = \int_{-2}^{x} f(t)\,dt.$$



(a) Evaluate $F(1)$.

(b) Evaluate $F(-3)$.

(c) Find $F'(-2)$.

(d) Find $F''(2)$.

(e) Find a value of x where $F(x)$ has a local minimum. **Justify your response.**

BASIC FORMULAS

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int e^u\,du = e^u + C$$

$$\int a^u\,du = \frac{a^u}{\ln a} + C$$

$$\int \sin(u)\,du = -\cos(u) + C$$

$$\int \cos(u)\,du = \sin(u) + C$$

$$\int \sec^2(u)\,du = \tan(u) + C$$

$$\int \csc^2(u)\,du = -\cot(u) + C$$

$$\int \sec(u)\tan(u)\,du = \sec(u) + C$$

$$\int \csc(u)\cot(u)\,du = -\csc(u) + C$$

$$\int \tan(u)\,du = \ln|\sec u| + C$$

$$\int \cot(u)\,du = \ln|\sin u| + C$$

$$\int \sec(u)\,du = \ln|\sec u + \tan u| + C$$

$$\int \csc(u)\,du = \ln|\csc u + \cot u| + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int f(u(x)) \cdot u'(x)\,dx = \int f(u)\,du$$

$$L_n = \Delta x \sum_{k=1}^{n} f(a + (k-1)\Delta x)$$

$$R_n = \Delta x \sum_{k=1}^{n} f(a + k\Delta x)$$