Exam 4 Review

1. Determine if the relation defines $y$ as a one-to-one function of $x$.
   a. $\{(−10, 4), (−2, 2), (6, 0), (14, −2)\}$
   
   b. 
   
   c.

2. Determine if the function is one-to-one. Give an explanation supporting your choice.
   a. $k(x) = x^3 + 8$
   b. $g(x) = x^2 - 6$
   c.
3. Use composition of functions to determine if the two functions are inverses.
   a. \( f(x) = \frac{4}{x-1} \) and \( g(x) = \frac{x+4}{x} \)

   b. \( r(x) = \frac{-2+x}{6} \) and \( s(x) = 6x - 2 \)

4. Given a one-to-one function find the inverse. Give your solution in inverse function notation.
   a. \( f(x) = 3\sqrt{2x - 5} \)

   b. \( g(x) = \frac{x-4}{x+2} \)

5. Graph the exponential function \( f(x) = 4^x \). Use a table to find points to plot. Write the domain and range of the function in interval notation.

6. Given the graph of \( f(x) = \left(\frac{1}{2}\right)^x \), graph \( g(x) = \left(\frac{1}{2}\right)^{x-3} + 2 \). State the shifts from the given graph.
7. Susan needs to borrow $15,000 dollars. She has 3 options to choose from:
   - 5.5% simple interest for 4 years
   - 5.25% interest compounded monthly for 4 years
   - 5% interest compounded continuously for 4 years
   a. How much total interest would Susan pay for each option?
   b. Which option would be best for Susan, assuming she wants to pay the least amount of interest possible.

8. A veterinarian depreciates a $10,000 X-ray machine. He estimates that the resale value $V(t)$ after $t$ years is 90% of its value from the previous year. Therefore, the resale value can be approximated by $V(t) = 10,000(0.9)^t$.
   a. Find the resale value after 4 years.
   b. If the veterinarian wants to sell his practice 8 years after the X-ray machine was purchased, how much is the machine worth? Round to the nearest $100.

9. Write the logarithmic equations in exponential form.
   a. $\log_5 125 = 3$
   b. $\ln(x + 4) = 3$
   c. $\log_K R = m$

10. Write the exponential equations in logarithmic form
    a. $2^4 = 16$
    b. $(\frac{1}{2})^{-5} = 32$
    c. $a^9 = b$

11. Evaluate the following logarithmic expressions without using a calculator. Show all work.
    a. $\log_2 64$
    b. $\log_3 \left(\frac{1}{9}\right)$
    c. $\log 10,000$
12. Simplify the following expressions
   a. $2^{\log_2 (x-y)}$
   b. $\log_7 7^{13}$
   c. $\ln e^{x^2+2}$
   d. $\log_p p$
   e. $\ln 1$

13. Identify the shifts from the graph of the “parent” function $f(x) = \log_2 x$ to the graph of the related function $g(x) = \log_2 (x + 5) - 2$.

14. Apply product, quotient and power properties of logarithms and simplify if possible
   a. $\log_4 \sqrt[5]{x^7}$
   b. $\log \left( \frac{m^2+n}{100} \right)$
   c. $\log_7 (49k)$

15. Write the logarithm $\ln \frac{x^2+4}{\sqrt{e^3}}$ as a sum or difference of logarithms. Simplify each term.

16. Write the logarithmic expression $4\log_6 r - 3\log_6 s - 2\log_6 t$ as a single logarithm with coefficient 1. Simplify.

17. Solve the following Exponential equations. Give an exact solution and an approximation to 4 decimal places as needed.
   a. $5^{3x-2} = 625$
   b. $6^{2x-5} - 8 = 12$
   c. $e^{2x} + 2e^x - 24 = 0$
18. Solve the following Logarithmic equations. Give an exact solution and an approximation to 4 decimal places as needed.
   a. \( \log_3(2x + 5) = \log_3(5x - 13) \)
   b. \( \log_6(7x - 2) = 1 + \log_6(x + 5) \)
   c. \( \log_2 x = 4 - \log_2(x - 6) \)

19. Matilda and Horace open a retirement account with $120,000. How many years will it take for the money to grow to $800,000 if it grows at a rate of 4.5% compounded monthly? Round to the nearest year.

20. The variable \( pH \) in the formula \( pH = -\log[H^+] \) represents the level of acidity or alkalinity of a liquid, and \( [H^+] \) (in mol/L) is the concentration of hydronium ions in the solution.
   a. Determine the \( pH \) of the following
      - Tomato with \( [H^+] \) of \( 6.31 \times 10^{-5} \) mol/L
      - Milk of Magnesia with \( [H^+] \) of \( 3.16 \times 10^{-11} \) mol/L
   b. Determine the \( [H^+] \) of the following
      - Onion with a \( pH \) of 5.6
      - Baking Soda with a \( pH \) of 8.4

21. The population of a rural community was 3050 in 2010 and the population had increased to 3500 in 2015. Assuming exponential growth find the following.
   a. The growth rate to 4 decimal places and as a percentage.
   b. Estimate the population in 2018 if the trend continues.

22. A sample found at a dig site was determined to have 68.7% of the C-14 still remaining. Use the model \( Q(t) = Q_0 e^{-0.000121t} \) to determine the age of the sample. Round to the nearest year.
23. Given the following data

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>2.4</td>
</tr>
<tr>
<td>1</td>
<td>3.5</td>
</tr>
<tr>
<td>2</td>
<td>5.5</td>
</tr>
<tr>
<td>3</td>
<td>8.1</td>
</tr>
<tr>
<td>4</td>
<td>12.0</td>
</tr>
<tr>
<td>5</td>
<td>18.4</td>
</tr>
</tbody>
</table>

Use your graphing calculator to find an exponential regression model that fits the data. Round to 3 decimal places.

24. A lake is stocked with bass by the U.S. Park Service. The population of bass in the lake is given by the Logistic Model

\[ P(t) = \frac{3000}{1 + 2e^{-0.37t}} \]

where \( t \) is the time in years after the lake was stocked.

a. Evaluate \( P(0) \) and interpret its meaning in the context of the problem.

b. Use the function to predict the bass population 4 years after the lake was stocked.

c. Determine the number of years for the bass population to reach 2800. Round to the nearest year.