INSTRUCTIONS: This exam is a closed book exam. You may not use your text, homework, or other aids except for a 3×5 inch notecard. You may use an allowable calculator, TI-83 or TI-84 to
• perform operations on real numbers,
• evaluate functions at specific values, and
• look at graphs and/or tables.
A TI-89, TI-Nspire, or a calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are not allowed. Unless otherwise stated, you must show all of your work including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that correct answers using incorrect reasoning may not receive any credit. Reasoning which will earn credit will use material covered in the course to date, up to and including Section 3.10 and Section 4.2.
Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 9 problems on 10 pages. Make sure all problems and pages are present.

The exam is worth 100 points in total.

You have 60 minutes to work starting from the signal to begin. Good luck!
### Exam 1 Grade by Problem Number

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### Current Course Grade by Category

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1. (3 points each) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

(i) The function $y = g(t)$ measures the amount of iron in Mikayla’s bloodstream (in milligrams) where $t$ is measured in minutes since she ingested an iron supplement. What is the most appropriate interpretation of $g'(28) = 3.7$?

a. The amount of iron in Mikayla’s bloodstream grew by 3.7 milligrams during the 29th minute after she ingested the iron supplement.

b. Twenty-eight minutes after ingesting the iron supplement, Mikayla had 3.7 milligrams of iron in her bloodstream.

c. Twenty-eight minutes after ingesting the iron supplement, the amount of iron in Mikayla’s bloodstream was growing at a rate of 3.7 milligrams per minute.

d. On average, the amount of iron in Mikayla’s bloodstream increased by 3.7 milligrams each minute over the first 28 minutes since she ingested the iron supplement.

e. The amount of iron in Mikayla’s bloodstream grew by 3.7 milligrams during the first 28 minutes after she ingested the iron supplement.

(ii) Suppose that $y = f(u)$ and $u = g(x)$ are differentiable functions of the input variables $u$ and $x$ respectively. The derivative of the composite function $y = [f \circ g](x)$ at the input value $x = 2$ is given by the formula

a. $[f \circ g]'(2) = f'(2)g'(2)$

b. $[f \circ g]'(2) = f' (g(2)) g'(2)$

c. $[f \circ g]'(2) = f'(2)g(2) + g'(2)f(2)$

d. $[f \circ g]'(2) = f' (g'(2))$

e. $[f \circ g]'(2) = f (g(2))$
(iii) Consider the function \( f(x) = x - \sqrt{x} \). Which of the following functions gives the average rate of change of \( f(x) \) with respect to \( x \) on the interval from \( x = 9 \) to \( x = 9 + h \)?

a. \( g(h) = \frac{9 + h - \sqrt{9 + h}}{h} \)

b. \( g(h) = \frac{9 + h - 9}{9 + h - \sqrt{9 + h} - 6} \)

c. \( g(h) = \frac{9 + h - \sqrt{9 + h} - (9 - \sqrt{9})}{h} \)

d. \( g(h) = \frac{3}{h} \)

e. \( g(h) = 1 + \frac{1}{2\sqrt{x}} \)

(iv) The equation of the line tangent to the graph of \( y = \ln(x) \) when \( x = 4 \) is

a. \( y = \frac{1}{4}(x - 4) + \ln(4) \)

c. \( y = \ln(4)(x - 4) + \frac{1}{4} \)

d. \( y = \frac{1}{x}(x - 4) + \ln(4) \)

e. \( y = \frac{1}{4}(x - 4) - \ln(4) \)

(v) The expression \( \lim_{h \to 0} \frac{2(x + h)^7 - 3(x + h) + 1 - (2x^7 - 3x + 1)}{h} \) is the derivative of what function?

a. \( f(x) = 2(x + h)^7 - 3(x + h) + 1 \)

b. \( f(x) = 2(x + h)^7 - 3(x + h) + 1 - (2x^7 - 3x + 1) \)

c. \( f(x) = 14x^6 - 3 \)

d. \( f(x) = 2x^7 - 3x + 1 \)

e. \( f(x) = \frac{2(x + h)^7 - 3(x + h) + 1 - (2x^7 - 3x + 1)}{h} \)
2. (8 points) Use the definition of derivative to show that \( \frac{d}{dx} (3x^2 - 4) = 6x \). You must use the definition of derivative to receive any credit.

\[
\lim_{h \to 0} \frac{3(x+h)^2 - 4 - (3x^2 - 4)}{h} = \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 4 - 3x^2 + 4}{h} \\
= \lim_{h \to 0} \frac{6xh + 3h^2}{h} \\
= \lim_{h \to 0} 6x + 3h \\
= 6x.
\]

3. (9 points) The figure below shows the line tangent to the graph of the circle \((x + 1)^2 + (y - 2)^2 = 9\) at the point \((1, \sqrt{5} + 2)\). Use calculus to determine the slope of this tangent line.

\[
2(x+1) + 2(y-2) \frac{dy}{dx} = 0 \\
\frac{dy}{dx} = \frac{-(x+1)}{y-2} \\
\frac{dy}{dx} = \frac{-2}{\sqrt{5}+2-2} \\
\frac{dy}{dx} = \frac{-2}{\sqrt{5}}.
\]
4. *(10 points)* Use calculus to determine the following for the function \( f(x) = \frac{1}{4} (x^3 - 21x - 12) \).

(a) *(5 points)* Find the critical points of \( f \). State your answers as exact values. (Work must be shown to receive credit.)

\[
\frac{f'(x)}{4} = \frac{3x^2 - 21}{4}
\]

\[
0 = \frac{3}{4} (x^2 - 7)
\]

\[
x = \pm \sqrt{7}.
\]

(b) *(5 points)* Determine the absolute maximum and absolute minimum of \( f \) on the interval \([-4, 5]\). Round your answers to two decimal places. (Work must be shown to receive credit.)

\[
f(-\sqrt{7}) =
\]

\[
f(\sqrt{7}) =
\]

\[
f(-4) =
\]

\[
f(-5) =
\]
5. (10 points) Given the graph of \( y = f(x) \), determine the following, if they exist. (Assume that the end behavior of \( f \) is as indicated.)

(a) (2 points) Determine \( f'(5) \).

\[
\frac{2-1}{6-4} = \frac{1}{2}.
\]

(b) (3 points) Determine all critical points of \( f \) on the interval \((-6, 6)\).

\[ x = -4, 2, 4 \]

(c) (5 points) Define \( g(x) = \frac{2x}{f(x) + 1} \) where \( f \) is as above. Find \( g'(-4) \).

\[
g'(x) = \frac{(f(x)+1)(2) - 2x(f'(x))}{(f(x)+1)^2}
\]

\[
g'(-4) = \frac{(f(-4)+1)(2) - 2(-4)f'(-4)}{(f(-4)+1)^2} = \frac{(3+1)2 - 2(-4)(0)}{(3+1)^2} = \frac{8}{16} = \frac{1}{2}
\]
6. (6 points) Consider the graph of \( y = f(x) \) illustrated below.

Write each graphical quantity, A-F, in the blank next to corresponding expression on the left. \textit{Each letter will be used exactly once.}

<table>
<thead>
<tr>
<th>Expression</th>
<th>Graphical Quantity</th>
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<tr>
<td>( h )</td>
<td>E</td>
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<tr>
<td>( f(a) )</td>
<td>A</td>
</tr>
<tr>
<td>( f(a + h) )</td>
<td>B</td>
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<td>( f(a + h) - f(a) )</td>
<td>F</td>
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<tr>
<td>( \frac{f(a + h) - f(a)}{h} )</td>
<td>D</td>
</tr>
<tr>
<td>( \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} )</td>
<td>C</td>
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\( y = f(x) \)

\( a \) \hspace{1cm} \( a + h \)
7. (5 points each) Compute the following derivatives. Do not simplify.

(a) Let $g(x) = 7x^3 - \pi x + \sin(x)$. Find $g'(x)$.

$$g'(x) = 21x^2 - \pi + \cos(x)$$

(b) Let $y = 5^x \cdot \ln(x)$. Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = (5^x \ln(5))(\ln(x)) + (5^x)(\frac{1}{x})$$

(c) Let $\theta = \cos^{-1}(\sqrt{x})$. Find $\frac{d\theta}{dx}$ in terms of $x$ only.

$$\frac{d\theta}{dx} = \frac{-1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2}x^{-\frac{1}{2}}$$

Since $\frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}$

(d) Let $f(t) = \frac{e^t}{\sec(t)}$. Find $f'(t)$.

$$f'(t) = \frac{\sec(t) \cdot e^t - e^t \cdot \sec(t) \cdot \tan(t)}{(\sec(t))^2}$$
8. 

\( (10 \text{ points}) \) Find \( \frac{dy}{dx} \) for the curve \( \cos(y) = y - x^2 \). You must solve for \( \frac{dy}{dx} \) in terms of \( x \) and \( y \).

\[-\sin(y) \frac{dy}{dx} = \frac{dy}{dx} - 2x\]

\[2x = \frac{dy}{dx} + \sin(y) \frac{dy}{dx}\]

\[2x = \frac{dy}{dx} (1 + \sin(y))\]

\[\frac{dy}{dx} = \frac{2x}{1 + \sin(y)}\]

9. 

\( (12 \text{ points}) \) A hot-air balloon rising straight up from a level field is tracked by a range finder 700 feet from the lift-off point (see the image below). At the moment the range finder’s elevation angle is \( \frac{\pi}{6} \), this angle is increasing at a rate of 0.16 radian per minute. How fast is the balloon rising at that moment?

\[
\text{given: } \frac{d\theta}{dt} = 0.16 \text{ when } \theta = \frac{\pi}{6}
\]

\[
\text{want: } \frac{dh}{dt}
\]

We need a relationship between \( \theta \) and \( h \).

\[
\tan(\theta) = \frac{h}{700}
\]

Take the derivative implicitly with respect to \( t \)

\[
\sec^2(\theta) \frac{d\theta}{dt} = \frac{1}{700} \frac{dh}{dt}
\]

\[
\sec^2 \left( \frac{\pi}{6} \right) (0.16) = \frac{1}{700} \frac{dh}{dt}
\]

\[
\frac{dh}{dt} = 700 \sec^2 \left( \frac{\pi}{6} \right) (0.16)
\]

\[
\frac{dh}{dt} = 149.33
\]