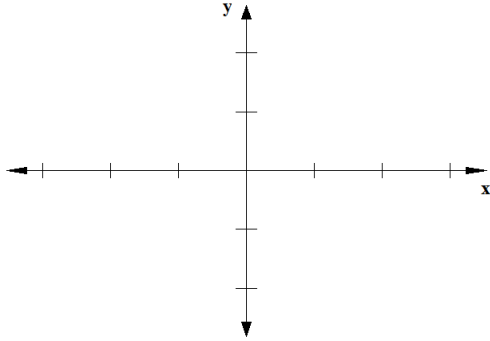


Name \_\_\_\_\_

**Calculus III**  
**Exam I Practice Problems**

8 points

1. Sketch  $\mathbf{v} = \langle 2, -2 \rangle$ ,  $\mathbf{w} = \langle -3, 1 \rangle$ ,  $\mathbf{v} + 2\mathbf{w}$  and  $\mathbf{v} - \mathbf{w}$ .



8 points

2. (a) Find a vector of length 26 in the direction of  $\mathbf{v} = 5\mathbf{i} - 12\mathbf{j}$ .  
(b) Find a unit vector making an angle of  $300^\circ$  counterclockwise from the  $+x$ -axis.

10 points

3. (a) Find parametric equations for the line that passes through  $(-1, 1, 0)$  and  $(2, 1, 4)$ .  
(b) Find the intersection of the lines  $\mathbf{r}_1(t) = \langle 0, 1, 1 \rangle + t\langle 1, 1, 2 \rangle$  and  $\mathbf{r}_2(s) = \langle 2, 0, 3 \rangle + s\langle 1, 4, 4 \rangle$

9 points

4. Find the force in Newtons that is required to push a 25-kg wagon up a  $12^\circ$  incline if the acceleration of gravity downward is  $g = 9.8 \text{ m/s}^2$ .

10 points

5. Find the angle between the vectors  $\langle 3, 1 \rangle$ ,  $\langle -2, 1 \rangle$ .

12 points

6. (a) Sketch the parallelogram formed using as two sides  $\mathbf{u} = \langle 1, -1, 1 \rangle$  and  $\mathbf{v} = \langle 1, 0, 1 \rangle$ , and find its area.  
(b) Calculate  $(\mathbf{j} - \mathbf{k}) \times (\mathbf{j} + \mathbf{k})$ .  
(c) Find the volume of the parallelepiped formed using as three edges sides  $\mathbf{u} = \langle 0, -1, 1 \rangle$ ,  $\mathbf{v} = \langle 1, 0, 1 \rangle$ ,  $\mathbf{w} = \langle 1, 1, 1 \rangle$  using the triple scalar product.

12 points

7. (a) Find the equation of the plane that passes through  $P(3, 5, -4)$  and is parallel to the plane  $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ .

- (b) Sketch the plane  $x + 3y + 2z = 6$  in the first quadrant.
- (c) Find the plane that contains the three points  $P(5,1,1)$ ,  $Q(1,1,2)$ ,  $R(2,2,1)$ .

15 points each

8. State the type of surface and sketch it with its trace in the  $z = 0$  plane for

(a)  $x^2 + 16y^2 - 4z^2 = 16$

(b)  $x - 9y^2 - 4z^2 = 6$

(c)  $z = y^2 - x^2 + 1$

16 points

9. (a) Find the vertices, foci, and center of the ellipse

$$\left(\frac{x-1}{4}\right)^2 + \left(\frac{y}{9}\right)^2 = 1$$

(b) Find an equation for the ellipse with foci  $(0,7)$ ,  $(0,1)$  and semi-minor axis length 4.

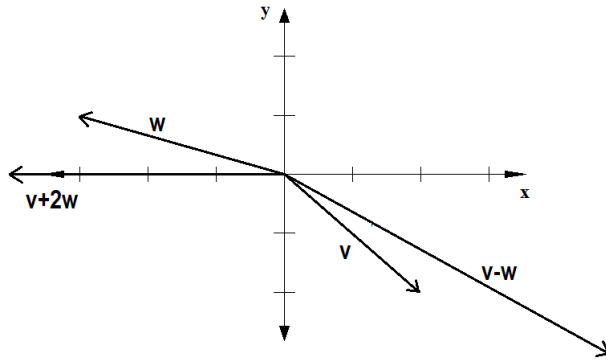
(c) Find the foci, and asymptotes of the hyperbola

$$4x^2 - 3y^2 + 8x + 30y = 215$$

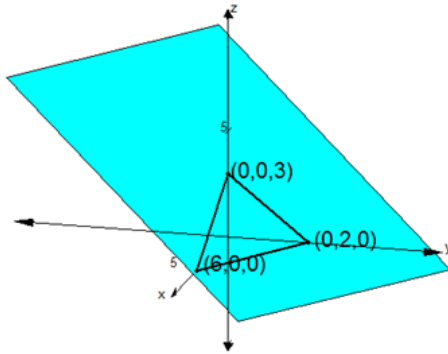
(d) Find the equation of the parabola with vertex  $(0,0)$  and focus  $(2,0)$ .

**Answers:**

1.



2. (a)  $10\mathbf{i} - 24\mathbf{j}$  (b)  $\frac{1}{2}\mathbf{i} - \frac{\sqrt{3}}{2}\mathbf{j}$
3. (a)  $x = -1 + 3t, y = 1, z = 4t$  (b)  $(3, 4, 7)$
4. 51 N
5.  $3\pi/4$
6. (a)  $\sqrt{2}$  (b)  $2\mathbf{i}$  (c) 1
7. (a)  $3x - 2y + z = -5$   
(b)



- (c)  $x + 3y + 4z = 12$
8. Sketches not included here...
- (a) Hyperboloid of one sheet,  $z = 0$  trace: the ellipse  $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{1}\right)^2 = 1$
- (b) Elliptic paraboloid,  $z = 0$  trace: the parabola  $x = 9y^2 + 6$
- (c) Hyperbolic paraboloid,  $z = 0$  trace: the hyperbola  $x^2 - y^2 = 1$
9. (a) center  $(1, 0)$ , foci  $(1, \pm\sqrt{65})$ , vertices  $(-3, 0), (5, 0), (1, 9), (1, -9)$
- (b)  $\left(\frac{x}{4}\right)^2 + \left(\frac{y-4}{5}\right)^2 = 1$
- (c)  $(-1, 15), (-1, -5), y - 5 = \pm \frac{2}{\sqrt{3}}(x + 1)$
- (d)  $y^2 = 8x$