Calculus III
Exam I Practice Problems

8 points
1. Sketch \( \mathbf{v} = <2,-2>, \mathbf{w} = <-3,1>, \mathbf{v} + 2\mathbf{w} \) and \( \mathbf{v} - \mathbf{w} \).

8 points
2. (a) Find a vector of length 26 in the direction of \( \mathbf{v} = 5\mathbf{i} - 12\mathbf{j} \).
   
   (b) Find a unit vector making an angle of 300° counterclockwise from the +x-axis.

10 points
3. (a) Find parametric equations for the line that passes through (-1,1,0) and (2,1,4).
   
   (b) Find the intersection of the lines \( \mathbf{r}_1(t) = <0,1,1> + t<1,1,2> \) and \( \mathbf{r}_2(s) = <2,0,3> + s<1,4,4> \).

9 points
4. Find the force in Newtons that is required to push a 25-kg wagon up a 12° incline if the acceleration of gravity downward is \( g = 9.8 \text{ m/s}^2 \).

10 points
5. Find the angle between the vectors \( <3,1>, <-2,1> \).

12 points
6. (a) Sketch the parallelogram formed using as two sides \( \mathbf{u} = <1,-1,1> \) and \( \mathbf{v} = <1,0,1> \), and find its area.

   (b) Calculate \( (\mathbf{j} - \mathbf{k}) \times (\mathbf{j} + \mathbf{k}) \).

   (c) Find the volume of the parallelepiped formed using as three edges sides \( \mathbf{u} = <0,-1,1>, \mathbf{v} = <1,0,1>, \mathbf{w} = <1,1,1> \) using the triple scalar product.

12 points
7. (a) Find the equation of the plane that passes through P(3,5,-4) and is parallel to the plane \( 3\mathbf{i} - 2\mathbf{j} + \mathbf{k} \).
(b) Sketch the plane \( x + 3y + 2z = 6 \) in the first quadrant.

(c) Find the plane that contains the three points \( P(5,1,1), Q(1,1,2), R(2,2,1) \).

15 points each

8. State the type of surface and sketch it with its trace in the \( z = 0 \) plane for
   (a) \( x^2 + 16y^2 - 4z^2 = 16 \)
   (b) \( x - 9y^2 - 4z^2 = 6 \)
   (c) \( z = y^2 - x^2 + 1 \)

16 points

9. (a) Find the vertices, foci, and center of the ellipse
   \[
   \left( \frac{x-1}{4} \right)^2 + \left( \frac{y}{9} \right)^2 = 1
   \]
   (b) Find an equation for the ellipse with foci \((0,7), (0,1)\) and semi-minor axis length 4.
   (c) Find the foci, and asymptotes of the hyperbola
   \( 4x^2 - 3y^2 + 8x +30y = 215 \)
   (d) Find the equation of the parabola with vertex \((0,0)\) and focus \((2,0)\).
Answers:

1. 

2. (a) $10 \mathbf{i} - 24 \mathbf{j}$  
   (b) $\frac{1}{2} \mathbf{i} - \frac{\sqrt{3}}{2} \mathbf{j}$

3. (a) $x = -1 + 3t$, $y = 1$, $z = 4t$  
   (b) $(3, 4, 7)$

4. $51$ N

5. $3\pi/4$

6. (a) $\sqrt{2}$  
   (b) $2\mathbf{i}$  
   (c) $1$

7. (a) $3x - 2y + z = -5$  
   (b) 
   (c) $x + 3y + 4z = 12$

8. Sketches not included here...

   (a) Hyperboloid of one sheet, $z = 0$ trace: the ellipse $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{1}\right)^2 = 1$

   (b) Elliptic parabolic, $z = 0$ trace: the parabola $x = 9y^2 + 6$

   (c) Hyperbolic paraboloid, $z = 0$ trace: the hyperbola $x^2 - y^2 = 1$

9. (a) center $(1, 0)$, foci $(1, \pm \sqrt{65})$, vertices $(-3, 0), (5, 0), (1, 9), (1, -9)$

   (b) $\left(\frac{x}{4}\right)^2 + \left(\frac{y-4}{5}\right)^2 = 1$

   (c) $(-1, 15), (-1, -5)$, $y - 5 = \pm \frac{2}{\sqrt{3}}(x + 1)$

   (d) $y^2 = 8x$