INSTRUCTIONS: This exam is a closed book exam. You may not use your text, homework, or other aids except for a 3 x 5-inch notecard. You may use an allowable calculator, TI-83 or TI-84 to
- perform operations on real numbers,
- evaluate functions at specific values, and
- look at graphs and/or tables.
A TI-89, TI-Nspire, or a calculator with a computer algebra system, any technology with wireless or Internet capability (i.e. laptops, tablets, smart phones or watches), a QWERTY keyboard, or a camera are not allowed. Unless otherwise stated, you must show all of your work including all steps needed to solve each problem and explain your reasoning in order to earn full credit. This means that correct answers using incorrect reasoning may not receive any credit. This exam assesses your understanding of material covered up to and including Section 5.7 of Rogawski and Adams.

Turn off all noise-making devices and all devices with an internet connection and put them away. Put away all headphones, earbuds, etc.

This exam consists of 8 problems on 11 pages. Make sure all problems and pages are present.

The exam is worth 100 points in total.

You have 60 minutes to work starting from the signal to begin. Good luck!
1. (3 points each) Answer the following multiple choice questions by circling your answer. No justification or explanation is required.

(i) The graph of a twice-differentiable function $f$ is shown below. Which of the following is true?

(a) $f(2) < f'(2) < f''(2)$
(b) $f'(2) < f(2) < f''(2)$
(c) $f''(2) < f'(2) < f(2)$
(d) $f(2) < f''(2) < f'(2)$
(e) $f''(2) < f(2) < f'(2)$

\[ f'(2) \text{ is neg} \]
\[ f(2) = 0 \]
\[ f''(2) \text{ is pos} \]

(ii) Suppose $f'(c) = 0$ and $f''(x) > 0$ for all $x$ near $c$. Which of the following is true?

(a) $f(c)$ is an inflection point.
(b) $f(c)$ is a local minimum.
(c) $f(c)$ is a local maximum.
(d) $f'(c)$ is positive.
(e) $f'(c)$ is negative.

(iii) Suppose $\int_0^1 f(x)dx = 5$ and $\int_0^7 f(x)dx = 1$. What is $\int_1^7 f(x)dx$?

(a) 5
(b) -4
(c) 6
(d) 4
(e) -6

\[ \int_1^7 f(x)dx = \int_0^7 f(x)dx - \int_0^1 f(x)dx \]
\[ = 1 - 5 \]
\[ = -4 \]
(iv) Consider the following graph of \( f(x) = x \sin(x) \) on the domain \([-4, 4]\). How many values of \( c \) in \((-4, 4)\) appear to satisfy the Mean Value Theorem equation
\[
f'(c) = \frac{f(b) - f(a)}{b - a}
\]
where \( a = -4 \) and \( b = 4 \).

(a) None  
(b) One  
(c) Two  
(d) Three  
(e) Four or more

(v) The expression \[
\frac{1}{10} \left( \left( \frac{1}{10} \right)^2 + \left( \frac{2}{10} \right)^2 + \left( \frac{3}{10} \right)^2 + \cdots + \left( \frac{20}{10} \right)^2 \right)
\]
is a Riemann sum approximation for which of the following expressions?

(a) \( \int_0^2 x^2 \, dx \)
(b) \( \int_0^2 \left( \frac{x}{10} \right)^2 \, dx \)
(c) \( \frac{1}{10} \int_0^2 \left( \frac{x}{10} \right)^2 \, dx \)
(d) \( \int_0^{20} x^2 \, dx \)
(e) \( \frac{1}{10} \int_0^2 x^2 \, dx \)

We are looking for points where the tangent line has the same slope as the line connecting \((-4, f(-4))\) and \((4, f(4))\).

We see the slope of that line is zero. So how many x-values in \([-4, 4]\) have a tangent line with slope 0?

The function we are plugging into \( \int \) is \( x^2 \), to get the heights.

\[
\int_0^2 x^2 \, dx
\]
2. (10 points) The following image illustrates a Riemann sum using \( N \) terms:

![Diagram of Riemann sum](image)

Write each item on the right in the blank next to the corresponding expression on the left. Items B, C, and \( A_3 \) refer to the labeled graphical quantities above. **Each item will be used exactly once.**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Matching Item</th>
<th>Item</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta x )</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>( f(x_3)\Delta x )</td>
<td>( A_3 )</td>
<td>C</td>
</tr>
<tr>
<td>( f(x_3) )</td>
<td>B</td>
<td>( A_3 )</td>
</tr>
<tr>
<td>( \sum_{i=1}^{N} f(x_i)\Delta x )</td>
<td>( A_1 + A_2 + \cdots + A_N )</td>
<td>( \int_a^b f(x)dx )</td>
</tr>
<tr>
<td>( \lim_{N \to \infty} \sum_{i=1}^{N} f(x_i)\Delta x )</td>
<td>( \int_a^b f(x)dx )</td>
<td>( A_1 + A_2 + A_3 + \cdots + A_{N-1} + A_N )</td>
</tr>
</tbody>
</table>
3. (5 points each) Evaluate the following limits or state that they do not exist ("DNE"). Use $\infty$ or $-\infty$ if either is appropriate. Numbers alone without justification (either algebra and/or quoting theorems where applicable) will receive no credit. If you use L'Hôpital's Rule, clearly indicate when you do so.

(a) $\lim_{x \to 0} \frac{\cos(x)}{x^2 + 1} = \frac{\cos(0)}{0^2 + 1} = \frac{1}{1} = 1$.

The function is continuous at 0.

(b) $\lim_{x \to 3} \frac{\sqrt{x + 1} - 2}{x^3 - 7x - 6} = \lim_{x \to 3} \frac{\frac{1}{2} (x+1)^{-\frac{1}{2}}}{3x^2 - 7} = \frac{\frac{1}{2} (3+1)^{-\frac{1}{2}}}{3 \cdot 3^2 - 7} = 0.125$

(c) $\lim_{x \to \infty} \frac{x^2 + 9x}{e^x} = \lim_{x \to \infty} \frac{2x + 9}{e^x} = \lim_{x \to \infty} \frac{2}{e^x} = 0$. 
4. (10 points) The graph of the function $f$ is shown below.

(a) (6 points) Use the above graph to fill in the following table with the signs of $f'(x)$ and $f''(x)$ on the indicated intervals. Use a “+” sign to indicate the values are always positive and a “−” sign to indicate the values are always negative.

<table>
<thead>
<tr>
<th></th>
<th>$(-6, -4.5)$</th>
<th>$(-4.5, -2)$</th>
<th>$(-2, 0)$</th>
<th>$(0, 2)$</th>
<th>$(2, 5)$</th>
<th>$(5, 6)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign of $f'(x)$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>Sign of $f''(x)$</td>
<td>−</td>
<td>−</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
</tbody>
</table>

(b) (2 points) List the $x$-coordinates of all critical points. (Note that there are two critical points in the interval shown on the graph above.)

$x = -2$

$x = 2$

(c) (2 points) List the coordinates of all inflection points.

$(-2, 2)$ and $(0, 3)$. 
5. (5 points each) Evaluate each of the following. Show all of your work and give exact answers. If the answer is a number, do not round.

(a) \[ \int (x - \cos(x) + x^{5/4}) \, dx = \frac{1}{2} x^2 - \sin x + \frac{4}{9} x^{9/4} + C. \]

(b) \[ \int_0^4 \sqrt{t} \, dt = \int_0^4 t^{1/2} \, dt = \frac{2}{3} t^{3/2} \bigg|_0^4 = \frac{2}{3} (4)^{3/2} - \frac{2}{3} (0)^{3/2} = \frac{16}{3}. \]

(c) \[ \frac{d}{dx} \int_0^{x^2} \tan(\theta) \, d\theta = \tan(x^2) \cdot 2x \quad \text{FToI and chain rule.} \]

(d) \[ \int x^3 \cos(x^4) \, dx = \frac{1}{4} \int \cos(u^4) \cdot 4x^3 \, dx = \frac{1}{4} \int \cos u \, du = \sin u + C \]
\[ u = x^4, \quad du = 4x^3 \, dx \]

\[ = \sin(x^4) + C. \]
6. (12 points) You create a box with no top (shown on the right) by cutting squares of side length $x$ out of a $9 \times 24$-inch piece of cardboard (shown on the left) and folding up the sides.

(a) (4 points) Define a function $V$ that gives the volume of the box in terms of side length $x$ of the square cutout.

$$V = x (9-2x)(24-2x) = 4x^3 - 60x^2 + 216x$$

(b) (8 points) Use calculus to determine the value of $x$ for which the box has maximal volume. **Justify your answer.**

$$V'(x) = 12x^2 - 132x + 216$$
$$0 = 12(x^2 - 11x + 18)$$
$$0 = (x-9)(x-2)$$
$$x = 9, 2.$$

Which critical point is a max?

$$V''(x) = 24x - 132$$

$$V''(9) = 24(9) - 132 \text{ positive so min}$$

$$V''(2) = 24(2) - 132 \text{ negative so max}$$

$V(x)$ has a maximum when $x = 2$ inches.
7. (10 points) The function $A$ is defined as $A(x) = \int_{-5}^{x} f(t)dt$. The graph of $f$ is shown below.

![Graph of f(x)](image)

(a) (4 points) Evaluate $A(-2)$ and $A(1)$.

\[
A(-2) = \int_{-5}^{-2} f(t)dt = \frac{1}{2}(3)(-4) = -6.
\]

\[
A(1) = \int_{-5}^{1} f(t)dt = -6 + (-12) = -18
\]

(b) (4 points) Find all values of $x$ on $[-5, 5]$ such that the graph of $A$ has a local minimum.

$A$ has a local min when $x=3$ because $\int_{-5}^{3} f(t)dt$ is the most negative area.

(c) (2 points) Find $A'(4)$.

\[
A'(4) = f(4) = 2.
\]
8. (4 points each) The table below shows the horizontal velocity, \( v(t) \), of a baseball (in feet per second) for various values of \( t \), which is the number of seconds elapsed since baseball was thrown.

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
<th>1.25</th>
<th>1.5</th>
<th>1.75</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v(t) )</td>
<td>132.0</td>
<td>130.03</td>
<td>128.08</td>
<td>126.16</td>
<td>124.27</td>
<td>122.41</td>
<td>120.58</td>
<td>118.78</td>
<td>117.0</td>
</tr>
</tbody>
</table>

(a) Use the right endpoint approximation with four subintervals, \( R_4 \), to approximate the total horizontal distance the ball traveled during the first two seconds after it was thrown.

\[
\Delta t = \frac{1}{2}.
\]

\[
R_4 = \frac{1}{2}(128.08) + \frac{1}{2}(124.27) + \frac{1}{2}(120.58) + \frac{1}{2}(117.0).
\]

\[
= 244.965 \text{ feet}.
\]

(b) Write an integral that represents the exact horizontal distance the ball traveled during the first two seconds since the ball was thrown. **Do not evaluate this integral.** *(Note: You do not need to construct a function definition for \( v(t) \). Simply use \( "v(t)" \)).

\[
\int_0^2 v(t) \, dt.
\]